Real interest rate risk in the argentine banking system. A measuring model

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Abstract

The exposure of the Argentine banking system to real interest rate changes is material and discourages long term credit. Quantification of this risk would help to manage it and may promote new credit, although it is not an easy job, especially in emerging markets. This paper proposes a Value at Risk (VaR) approach that uses Monte Carlo simulation. We estimate time series models (autoregressive with mean reversion and jumps) of the behavior of bank deposits and of the rate of inflation, attempting to keep them tractable for a local practitioner. Results show that short term funded banks would face more risk from inflation indexed claims than from nominal claims (and would therefore apply a greater premium for that risk, according to a risk adjusted return on equity approach – RAROC). This can be linked to the discussion on the “puzzle” of the relatively low use of indexation. The extent of the risk and the fact that the sign of the gap is the same across banks does not contribute to the development of derivative contracts. Results may also indicate distortions introduced by capital requirement regulations and accounting rules. A generalization of the methodology may be explored within the framework of Pillar II of Basel II.

JEL: C22, E37, E47, G12, G21, G28, G32

I. Introduction and goal of the paper

The Argentine banking system is exposed to real interest rate risk, partly as a consequence of the extreme crisis that took place at the end of 2001 and in 2002. A significant amount of the banking system’s assets are adjusted by the consumer price index, while liabilities do not follow that evolution, in addition to being shorter termed. Uncertainty about future real interest rates together with the high level of exposure bring about the perception of high risk and discourage the generation of risky assets such as long term loans to the private sector. Quantification of this risk, which is the main goal of this paper, would help to manage it and may potentially encourage long term loan granting.

Widely accepted methodologies to measure financial risks have been developed in recent years, such as Value at Risk (VaR). There are obstacles in applying these measures locally, mostly originated in discrepancies between the dynamics of domestic markets and the assumptions underlying the methodologies, or the requirements for their application.

This paper tries a methodology to measure real interest rate risk that overcomes these problems, without underestimating them, and which, at the same time, remains tractable for the risk analyst in the domestic market. A large number of theoretical and empirical developments on the behavior of interest rates have been studied (and are cited as reference), whose implementation implies a level of complexity and an investment of time amply exceeding those available to a banking analyst, especially in emerging markets, and are more appropriate for

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1 The author thanks Hernán Lacunza, Lorena Garegnani, Laura D’Amato, Gastón Repetto, Ricardo Bebczuk and Alejandra Anastasi for their valuable comments. The remaining errors are the author’s exclusive responsibility. The opinions are those of the author and do not necessarily represent those of the Banco Central de la República Argentina.
academic researchers. In this paper, in contrast, we look for a relatively-simple-to-develop model, without losing rigor.

Additionally, the topic of the paper is connected to issues which are relevant for the local current situation, such as the study of incentives for the use of indexation and possible distortions introduced by regulation.

Section II expands on the definition and management of real interest rate risk. It is explained there that an important factor is whether losses are immediately reflected in the banks’ books. Section III provides an account of the situation of the Argentine banking system. Then we turn to risk quantification. Section IV explains the concept of VaR and related technical aspects, the benchmark portfolio whose risk is going to be quantified and the selection of risk factors. As there are different methodologies to compute VaR (section V), their applicability to the case under study will be examined: first, the linear parametric method, or “Delta Normal”, then the Historical Simulation method and finally the Structured Monte Carlo. We will put forth reasons for the last one and, using it, we will obtain VaR results (section VI). With these results, the impact of risk on lending rates and new loans generation will be analyzed, using a RAROC approach (risk adjusted return on capital) (section VII). Section VIII explains certain risk premiums that have not been accounted for in the approach and possible research extensions. Additionally, the results are related to the discussion on the seeming puzzle of the scarce development of indexed securities. Section IX concludes, remarking on the impact of long term finance markets and the development of derivative markets.

II. Definition and management of real interest rate risk

Banks face the risk that their economic condition may be impacted by adverse fluctuations in market interest rates when the sensitivity of their assets to those changes differs from the sensitivity of their liabilities. Some essential characteristics of bank operations expose them to interest rate risk, such as receiving deposits that are shorter termed compared to the loans they grant.

A risk that is akin to interest rate risk arises when index adjustment is applied on capital. If a bank’s asset sensitivity to changes in the adjustment index is not the same as that of its liabilities, there will be a risk due to this mismatch.

Fluctuations in interest rates and other financial variables have an immediate impact on the portion of banks’ portfolios that is registered at market values. In the same fashion, there is an impact on the economic value of the portion of bank’s portfolios whose accounting values do no follow market values, even when this impact does not show immediately but, instead, is reflected in the books progressively as future margins are recorded in financial statements. Banks take into consideration market risks, whether they are revealed in financial statements immediately or progressively, and they make decisions consistently with the risks arising from both types of recording.

A simple example can show the different types of recording. Let us assume a four-year-maturity loan of $100, with monthly amortizations, adjustable capital (linked to inflation) and a 6% interest rate on adjusted capital. Initially, there is a 10% annual rate of inflation (constant) a funding interest rate of 4.5% (also constant). Table 1 shows accounting values for outstanding capital and interest margins, when the variables remain at their original levels and when the funding rate jumps to 5.5% at moment 0 and remains at this higher level thereafter.

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2 To go deeper into the subject of interest rate risk in banks, an explanation of reduced technical difficulty can be seen, for example, in Bessis (2002).
The last line shows that the instantaneous impact on the economic value of the loan (present value of future cash flows) is 2%. If the claim was marked to market, the value of the loan would instantaneously show the loss; as time passes, the asset return would show collected interest plus parity evolution, which would tend to 100% towards maturity. If the claim is not marked to market, the loss stemming from the rate increase will not have immediate manifestation: it will be progressive, as paid and collected interest is accounted for.

### Table 1/ Example of the impact of an increase in the funding interest rate on an adjustable loan, accounting manifestation vs economic value

<table>
<thead>
<tr>
<th>months</th>
<th>outstanding capital</th>
<th>capital amortization</th>
<th>collected interest</th>
<th>paid interest</th>
<th>interest margin</th>
<th>paid interest</th>
<th>interest margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 12</td>
<td>82,9</td>
<td>26,4</td>
<td>5,7</td>
<td>4,3</td>
<td>1,4</td>
<td>5,2</td>
<td>0,5</td>
</tr>
<tr>
<td>13 to 24</td>
<td>61,0</td>
<td>29,2</td>
<td>4,5</td>
<td>3,4</td>
<td>1,1</td>
<td>4,2</td>
<td>0,4</td>
</tr>
<tr>
<td>25 to 36</td>
<td>33,7</td>
<td>32,2</td>
<td>3,1</td>
<td>2,3</td>
<td>0,8</td>
<td>2,8</td>
<td>0,3</td>
</tr>
<tr>
<td>37 to 48</td>
<td>0,0</td>
<td>35,6</td>
<td>1,3</td>
<td>1,0</td>
<td>0,3</td>
<td>1,2</td>
<td>0,1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>123,4</td>
<td>14,6</td>
<td>11,0</td>
<td>3,7</td>
<td>13,4</td>
</tr>
</tbody>
</table>

Present value of future cash flows (economic value) 125,7 123,2

Market risk management usually includes entering into hedging contracts, handling of long and short positions, and supporting unexpected risks with capital, in which case the satisfactoriness of the resulting return on capital ratio is assessed (economic capital approach). The supervisor, in turn, carries out a similar analysis, but with his own level of risk tolerance, which considers systemic factors and the protection of depositors. Regulation should create the right incentives to avoid interfering with return/risk analysis and, additionally, allow for the control of the level of risk that banks are assuming and how they are covering it.

A fundamental element to manage market risk is its quantification. In recent years, different measures of financial risks have been studied in the academic world and in the banking industry. Among them, those that calculate the worst expected loss, as a function of the historical behavior of relevant variables, are widely applied. A widely used measure of this type is Value at Risk or VaR. VaR measures are probabilistic and, like all statistical measures, are based on the characterization of the probability distribution of the variable under study, based on its history.

Thus, a VaR approach does not consider explicitly macroeconomic fundamentals such as real and monetary factors or issues related to technological changes that affect real interest rates. However, this does not mean that there is no control of reasonability on the results of models and forecasts, including the comparison with macroeconomic forecast models. In applying this approach to this paper, additionally, a quite peculiar historical period will be used.

### III. Risk management and the domestic situation

This paper studies the risk that fluctuations in banks’ funding rates should not be accompanied by variations in the index used to adjust the capital of certain claims, namely the CER index.

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3 The problem of interest rate mismatch is one of solvency (the fall of assets value) rather than one of liquidity (the need to have access to funding). A bank may resolve its liquidity risk by means of a credit line with maturity equal to that of the asset, but interest rate risk would persist if that line carried a floating rate. Consequently, it is right to deal with this risk with capital adequacy regulations.
Motivation stems from some singular characteristics of this mismatch: (i) it is significant; (ii) it has been persistent, (iii) it is a discouraging factor for new credit granting; and (iv) the risk is difficult to quantify.

The origin of the mismatch is important as it affects the possibility of managing it. Financial entities hold in their portfolios more than Arg$50 bn in sovereign debt, denominated in pesos and adjusted by CER, which represents around 60% of their total sovereign debt positions and 1.8 times their Net Worth (as of June 2006). Within this group, Bonos Garantizados (BOGAR) show the highest amount, with more than Arg$22 bn, followed by Préstamos Garantizados (PG), for around Arg$18 bn. Holdings of CER adjusting Central Bank bonds are around Arg$6 bn and then there are holdings derived from the latest swap of National Government debt in default and adjusting BODEN holdings, both totaling slightly over Arg$2 bn. Other bonds and claims add up to another Arg$2 bn.

Holdings of adjusting bonds, namely PG, BOGAR, BODEN and bonds from the swap of sovereign debt in default, which in all represent 80% of adjusting claims in the portfolios, have mainly originated as part of specific processes. PG were issued in the context of a voluntary swap of sovereign debt in late 2001. Then, in 2002, dollar denominated PG were converted into pesos at a 1.40 Arg$/ USD rate and their principal began to be adjusted by CER. There is a large diversity of PG, some of them with residual terms that exceed 25 years. BOGAR bonds originated in a “provincial Debt swap” in 2002, when provincial debt was converted into national debt, and expire in 2018. BODEN bonds were an instrument used in the resolution of the 2001/2002 crisis. Adjusting BODEN granted to the banks to compensate for “asymmetrical pesoization” fall due in February 2007, thus their outstanding amount has decreased rapidly. Bonds originated in the swap of defaulted debt have maturity dates later than 2030, although their amount is less significant in banks’ positions.

These special processes were accompanied by the introduction of regulatory benefits attached to the securities that the banks received, concerning prudential requirements and valuation rules. These regulatory benefits have progressive expiration schedules but, as long as they subsist, an incentive to keep positions is implicit, for example, due to the fact that institutions should in general show losses if they sold their holdings.

On the other hand, banks have adjusting liabilities for a smaller amount (mainly rediscounts with BCRA, which have decreased strongly and, to a lesser extent, new CER adjusting deposits). These generate a mismatch which is mainly funded with deposits, whose return follows market interest rates (time deposits).

In recent months, expectations of a relatively high inflation rate have encouraged banks to keep their holdings in CER-adjustable securities, as they offered a higher short term return, compared with other investments.

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4 The Central Bank constructs the CER coefficient using the Consumer Price Index (IPC), which is computed and released by the Instituto Nacional de Estadísticas y Censos – INDEC-, the National Institute of Statistics and Census.

5 Figures for this and the next paragraphs have been taken from the Superintendency of Financial Institutions (Superintendencia de Entidades Financieras y Cambiarias, SEFyC, Análisis del Sistema – 2006).

6 The National Government handed out these bonds to financial entities to compensate for net worth losses stemming from the compulsory conversion to pesos of banks’ loans denominated in dollars, at a 1 Arg. $ / USD ratio, while a conversion rate of 1.40 Arg. 4 /USD was imposed on deposits.

7 Mainly contained in Central Bank communiqué Com. “A” 3911 and its amendments. See BCRA, ordered texts.

8 As of January 2007, only two banks have liabilities from rediscounts with the BCRA.
The amount of the mismatch has been significant since the crisis and has not shown a decreasing trend so far. Its evolution for the banking system as a whole is depicted in Graph 1\(^9\). The mismatch was around Arg$30bn by mid 2006.

Usually banks do not assume risk from mismatches as large as those currently faced by the Argentine banking system, but they handle mismatches using hedging financial products, or passing on part of their assets (and risks) to the market through securitizations. These possibilities are limited in Argentina for several reasons; among them, the existence of regulatory benefits is important, as has been explained. As for the liability side, banks have clear motivations to pay off rediscounts with the Central Bank, which go beyond the management of interest rate mismatches and which are related to bank’s reputation, regularization plans, etc. In addition, hedging markets are not sufficiently developed.

Given the mismatch, the economic value of the banks’ portfolios would suffer a loss if the real interest rate should increase, that is, should there be an increase in the nominal interest rate which was not accompanied by an increase in inflation (as captured by CER), or should inflation be less than expected, and this did not come with a decrease in nominal interest rates. As long as assets producing the mismatch are not recorded in the banks’ books for their economic value, or market value, accounting figures will not show these fluctuations immediately, but they will, eventually, as fluctuations in real interest rates impact interest margins.

Not only are there difficulties to hedge mismatches but also to quantify implicit risk. This is a consequence of the fact that financial risk methodologies use statistical methods which, in general, rely on historical time series of the variables, and therefore it is important that these series refer to regimes and economic/financial environments which are similar to those foreseeable for the future that is forecast. Given structural changes that have taken place in the country in different periods, it is especially difficult in Argentina to obtain time series that are both relevant and sufficiently long. Additionally, even in developed countries, markets for indexed securities are relatively less developed, and the assessment of related risks has not

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received so much attention from risk specialists\textsuperscript{10}. Methodologies to assess nominal interest rate risk are not straightforwardly applicable to real rates, given that the adjusting index is applied on capital, and it is not an interest rate, and therefore risk models need to undergo certain adaptations.

The financial situation has also been unusual lately, in that banks’ funding real interest rates have been very low or negative, something not seen for 15 years (Graph 2). Although it can be expected that the real interest rate will tend to become positive, in order to measure risk we do not focus only on expected values, but on extreme events that can take place as a result of shocks, or unexpected events.

\textit{Graph 2. Real Interest Rate}

\textit{(using short term time deposit interest rate and monthly inflation rate)}

It is worth noting that in the last 15 years, the period until the end of 2001 was characterized by a Convertibility regime, or currency board, which achieved low inflation rates as compared to previous periods, but which is also consistent with higher volatility of interest rates due to the limitations it imposes on monetary policy, as compared to more flexible monetary regimes. The abandonment of Convertibility by late 2001 was followed by a very high instability period of around 18 months.

Current monetary and floating currency policies would in principle lead to less expected volatility of market interest rates and better absorption of exogenous shocks. However, monetary policy is not fully functional yet due to the lack of depth of financial markets and the limited participation of credit in the economy. This leads to think that there are still certain aspects of the current financial situation that are temporary.

\textit{IV. Value at Risk (VaR)}

VaR is a category of risk measures which is used to get a critical loss in the market value of a portfolio\textsuperscript{11}.

\textsuperscript{10} There are of course cases, such as Shen (1988) and Kothari et al. (2004).

\textsuperscript{11} To go deeper into market risk VaR see Jorion (2001), Holton (2002) and Mina et al. (2001).
The market value of a portfolio is known at present but it is uncertain in the future. This end-of-period value is a random variable with a probability distribution, conditional on available information at moment 0. Market risk of the portfolio can be quantified using a description of that conditional distribution which, according to the technique employed to compute VaR, will be complete or incomplete (if the whole distribution or some of its parameters are estimated, respectively). Quantiles of portfolio loss distributions are the most widely used measures of VaR. Thus, if the 0.95 quantile of the daily loss distribution of a portfolio is $1 million, that will be the figure for the 5% VaR, which means that it is expectable that the portfolio may lose less than $1 million in 19 out of 20 days or, equally, that it may lose more than that amount in 5% of the days. VaR assumes that the portfolio stays invariant during the period.

When many securities make up a portfolio, mapping functions are sought to make the measurement of VaR more operative, that is, functions that map securities’ values into basic risk factors, such as interest rates, currencies and commodity prices. A mapping function characterizes the sensitivity of portfolio’s components’ values to changes in risk factors by means of relations that are usually not linear. Then, a portfolio’s value is a linear function of the value of its components, as it is the sum of each position, weighted by the corresponding values. The mostly widespread mapping approach for portfolios that basically depend on interest rates is that of Modified Duration (MD). This approach allows for a first order, or linear, approximation to the portfolio’s value. By studying the conditional distribution of the interest rate on which the portfolio depends, its maximum variation can be estimated, with a given level of confidence and a given time horizon. Then, this “critical” change in interest rate is translated into a change in portfolio’s value, according to the mapping function.

Formally, when a portfolio (P) is expressed as a function of the prices of component assets, the portfolio is a linear polynomial of those prices. Variance of portfolio’s return $R_P$ is

$$Variance\left(R_P\right) = \sigma^2_P = x^\top \Sigma x,$$

where vectors $x$ reflect positions and $\Sigma$ is the variance-covariance matrix of returns of component assets. From that, and assuming conditional normality of the distribution of $R_P$ at moment 1, the most widely used expression of VaR is derived, according to which

$$VaR = \alpha \sigma_p,$$

where $\alpha$ is the “z-value” in the standard Normal distribution corresponding to the chosen confidence level and $\sigma_p$ is the standard deviation of portfolio’s return according to the previous expression. Volatilities must refer to returns that are measured on a consistent basis with VaR time horizon. If this is not the case and under the assumption of return independence, volatility can be scaled using the square root of time rule. Therefore, the volatility of returns for a $T$ day horizon is estimated as the daily return volatility times the square root of $T$, and the previous expression is changed into:

$$VaR = \alpha \sigma_p T^{0.5}.$$

When there is a mapping, the expression of the portfolio as a function of risk factors is usually no longer a linear relation. Portfolio’s value turns to be expressed as a function of (i) the risk factor (or a vector of risk factors) which, in the case under study, is the real interest rate or alternatively the nominal interest rate and the rate of inflation, and (ii) the function that maps the value of the portfolio into the risk factor(s).

When the mapping process produces non-linear portfolios, VaR calculation becomes more complex. There are a number of solutions to this, which include:

a) to approximate the desired quantile by using a Monte Carlo methodology;
b) to approximate the function to a linear polynomial (re-mapping) and after that to apply the linear solution;

c) to assume that risk factors are joint-normal and to apply appropriate probabilistic techniques for quadratic polynomials.

The simplest way to apply solution b) is by using the gradient of the non-linear function (first order of a Taylor polynomial) while a slightly more precise version would be the use of the gradient and the Hessian of the non-linear function (second order of a Taylor polynomial). In applying a MD approach in this study, we use a first order, or linear, approximation.

**IV.1. Selected variables and portfolio**

We are not interested in measuring a real portfolio’s VaR, but rather that of a hypothetical benchmark portfolio with Arg$100 initial nominal value. It is assumed that it is a 48-month loan, with monthly equal amortizations, principal adjusted by CER and monthly payments of a fixed interest rate. This loan is marginally funded, i.e., it is funded with new liabilities, which are assumed to be raised by the creditor institution as short-term time deposits. This allows the analysis to be made with respect to the interest rate of these deposits, rather than to an average funding rate. The idea is that there is already a portfolio of assets and their corresponding funding liabilities and, then, the addition of a marginal asset to this portfolio is analyzed.

Alternatively, it could have been put forward that the marginal funding was based on term deposits that pay the BADLAR\textsuperscript{12} rate. Results would not have changed significantly, since BADLAR and up-to-59-day deposit rates show similar behaviors in those aspects that are relevant for the analysis (volatility and correlation with the inflation monthly rate). Graph 3 intuitively shows this point. A higher volatility can be seen for private bank BADLAR but no clearly higher correlation with inflation, as compared to time deposit interest rates. The alternative use of the BADLAR rate would have posed additional difficulties derived from the fact that the monthly historical series is short and, although it could have been estimated backwards, it would anyway have been shorter than that of time deposits.

**Graph 3/ Recent evolution of 30-to-59-day deposit interest rates, private bank BADLAR and monthly inflation rate**

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\textsuperscript{12} BADLAR rate is the average interest rate of 30 to 35 day time deposits for one million pesos or more. It is calculated on average for the whole system and for private banks.
The definition or the real interest rate is not trivial. Applying Fisher’s equation:

\[(1+\text{real interest rate}) = (1+\text{nominal interest rate})/(1+\text{inflation rate}).\]

Thus, we could either work with the real interest rate, as a single risk factor, or take both the nominal interest rate and inflation as two risk factors.

The next decision concerns the use of ex post (observed) or ex ante (expected) interest rates. In this exercise we will work with ex post variables, given the obstacles to generating historical series of expected variables and given that it is the usual way to implement VaR techniques.

It is also pertinent to establish the frequency with which the above mentioned variables will be measured and expressed. For example, the inflation rate may measure the change in this month’s price index, or its change in the last 12 months, or the expected change for the coming 12 months, or other possible definitions. Specifically, we will work with the series for up to 59 day deposit interest rates, published by the BCRA and the monthly changes in the consumer price index, IPC.

It is very important to note, for this study, that the inflation rate in its annualized monthly change version shows changes that are usually significant but temporary. Graph 3 shows that rate as from July 2003. It can be seen that monthly or quarterly changes in this rate may be significant and are not permanent.

V. Different methods to compute VaR. Applicability

There are three traditional methods to calculate VaR: the parametric linear or “Delta-Normal” method, the Historical Simulation and the Structured Monte Carlo methods. In this study, in addition, the parametric method could be used in two versions: using the real interest rate as a single risk factor and using the inflation rate and the nominal interest rate as two risk factors.

V.1. “Delta Normal”, parametric model

The value of a financial asset (or a portfolio of them) is the present value of expected future cash flows (abstracting from considerations of risk, which will be discussed below). If an asset’s value depends essentially on interest rates, it is relevant to know what these rates will be along the life of the asset. Usually, and for the sake of simplicity, approaches such as the Modified Duration (MD) approach are applied, approximating the change in the asset’s value given a change in the asset’s internal risk or return (IIR) and making the following assumptions: (i) there is only one relevant interest rate—the market IRR for the duration of the asset-, or the yield curve is flat at that level of rates; (ii) movements in the yield curve are parallel; (iii) those movements are permanent; (iv) “coupon effects” are ignored – i.e. the different timing of cash flows between assets with the same duration and (v) the relationship between interest rates and the asset’s value is assumed linear, when it is not. All these simplifications substitute for the estimation of projected interest rates for the life of the asset.

Let us take first the Delta Normal method with one risk factor. For this paper the real interest rate is the risk factor. To measure the sensitivity of an inflation linked, fixed coupon rate asset’s value to changes in the real interest rate, the expression of the Modified Duration to changes in that rate should be derived. It is shown in Annex 1 that the MD to changes in the real interest rate has the same expression as MD for nominal assets. VaR is given then by the following expression:
\[ \text{VaR} = V \times \text{MD} \times \sigma \times 2.326 \quad (4) \]

where \( V \) stands for the asset’s present value, \( \sigma \) is the standard deviation of quarterly changes in the real interest rate, or “volatility”, and 2.326 is the “z-value” corresponding to a 99% confidence level in the standard Normal distribution. Percentage VaR is the same expression as before, not multiplying by the position. If changes in real interest rates are measured on the VaR time horizon, there is no need to multiply by the square root of time.

If, instead, we take the Delta Normal methodology with two risk factors, namely, the nominal interest rate and the inflation rate, and also applying a MD approach, we should map the position in the two risk factors. In the previous formula for VaR, the volatility will be obtained from the variance of the portfolio, which stands as

\[ \text{Varianza}(R_p) = \sigma_p^2 = x' \Sigma x, \quad (5) \]

The \( x \) vectors now reflect positions that are weighted by the sensitivity to the risk factors, whose variability are factored in through the variance-covariance matrix. In this exercise, components of \( x \) will be economical values multiplied by MDs to changes in the nominal interest rate and the inflation rate, respectively. The theoretical expression of MD to changes in inflation has no simple expression and can be alternatively measured “effectively”, that is, as the proportional change in the asset’s calculated value, given small changes in the inflation rate. For the asset in our example, this MD is around 2.

\textbf{V.1.a. Assessment of this approach to real interest rate risk}

For this paper we have concluded that working with the Delta Normal method, in its two options, would be wrong, given that certain assumptions of the methodology are not fulfilled and that certain data are not available. Specially problematic are the lack of a time structure of the relevant interest rates and the assumptions regarding the independence of changes in interest rates and constant volatility. As has been extensively shown in literature (a summary of which is included as Annex 4 of this paper) and in behavior models for the Argentine case, both assumptions are far from being supported by empirical observations. Instead, it is observed that:

- interest rates show a significant propensity to return to a long run average level, so that applying a Delta Normal method would overestimate risk;
- volatility changes in time and there are jumps in the behavior of the variables, so that the estimation of risk would not be appropriate.

It is worth remembering that the VaR concept and particularly its Delta Normal version have been developed to be mainly applied to equity portfolios, with very short time horizons (one day or a few days)\(^\text{13}\). The behavior of equity for that kind of horizon is usually characterized by stochastic processes that are different from those shown by interest rates, and which lead to the usual assumption that returns follow a random walk, which may be acceptable for equity prices and short horizons but not for interest rates and long horizons\(^\text{14}\).

It is important to point out that in the case of measuring VaR for real interest rate risk, inflation and interest rate future paths are relevant not only until the time horizon of the VaR (which in this paper is fixed at 3 months), but until the maturity of the portfolio. If the benchmark asset

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\(^{13}\) Such as in Riskmetrics, see Mina et al. (2001).

\(^{14}\) When a variable follows a random walk, the distribution of its changes is Normal and from this fact comes the name of the “Delta-Normal” approach.
expires in four years, its valuation and risk assessment would demand forming an expectation on interest rates and inflation paths for the next four years. But working this way would imply a simulation approach, which will be implemented later in this paper. With a Modified Duration approach, a measure of sensitivity would be multiplied by a change in the relevant rate of return for the duration of the asset. That being the case, a time structure of returns (or funding rates that banks pay for different terms) is required in order to analyze its historical volatility. Unfortunately, such data are not available.

In an attempt to ease the problem derived from the lack of such a time structure, we could try to build an interest rate for the term of the asset, building a historical series of real interest rates for the following two years (i.e. for the duration of the benchmark asset), accumulating monthly rates. This would imply assuming that long term rates, which are not observable, perfectly predict short term rates that will occur in the future and that there is no need for further information. Subsequently, a quarterly volatility for this historical series could be calculated. The existence of jumps and the propensity to return to the mean would be embedded in this calculation. But this ad-hoc technique would mean departing from the traditional parametric approach, in addition to demanding for the above mentioned strong assumption, and therefore, we would rather not embark on it.

V.2 Historical Simulation Approach

This technique takes historical data as a realization of the real process and characterizes the distribution of the risk factors according to that realization. This methodology was widespread in the mid ‘90s and has progressively lost popularity as techniques to generate pseudo-random numbers have become more accessible and consequently the implementation of Monte Carlo simulation.

Graph 4 shows the evolution of quarterly changes in the short term real annualized interest rate. Changes can be very abrupt from month to month, mainly as a result of changes in the inflation rate.

*Graph 4 / Quarterly changes in the real interest rate*
This methodology is also turned down for this paper, as using the worst quarterly changes of the real interest rate would ignore their temporariness.

**VI. Monte Carlo Simulation Method**

The Monte Carlo simulation method produces realizations of the variables using time series analysis techniques and generating random paths in order to be able to characterize the probability distribution of the variable of interest.

In order to apply this method, the evolution of the monthly time deposit interest rate and the monthly IPC index have been modeled employing widespread econometric time series models. The presence of structural changes forces calibration of behavior models on a relatively scarce set of data or, alternatively, on a long series, controlling for the most important changes. In our case, the models have been calibrated according to historical behavior for quite a long period in Argentina (from 1992 until the present), capturing specific periods by means of dummy variables.

By employing the estimated behavior models, Monte Carlo technique allows the simulation of a large number of probable paths (runs) for time deposit interest rates and IPC, during the assumed life of the contract. Based on each path, monthly payments are determined (amortization and interest) and, from them, the asset’s present value for each run. A distribution of present values is thus obtained and VaR can be determined by measuring the corresponding quantile in this distribution.

Modeling the dynamic of risk neutral rates is not necessary, given that our purpose is to compute VaR, and thus we work and project real variables. If the aim of the exercise was to value derivatives on these underlying variables, such an adjustment should be made.

A simulation technique is proposed considering that:

- Simulating IPC and interest rate evolutions separately is more appropriate, given that CER is not an interest rate but an index to adjust principal, and thus its financial effect is different (it is not paid off periodically but capitalized).
- It has been explained that the parametrical exercise would suffer from several methodological problems, in addition to the fact that the distribution of real interest rate changes approximates a Normal distribution imperfectly, something that would also introduce distortions.
- Outcomes depend on the interaction of projected variables’ initial levels (CER and time deposit interest rates), on their equilibrium levels, on the speed with which these variables tend to long term levels, on the probability of jumps, on volatilities and correlations.
- All these suggest a method that captures dynamic effects.

The models used to describe the evolution of the inflation rate and of the time deposit interest rate are time series econometric models and have been calibrated for the period April 1992 – August 2006.

**VI.1. The model for the time deposit interest rate**

**VI.1.a. Stylized facts of the short term time deposit interest rate.**
Table 2 shows summary statistics for the 30-59 day deposit interest rate, its logarithm and the difference of its logarithm, for the period from April 1992 until August 2006. Leptokurtosis is notable.

<table>
<thead>
<tr>
<th></th>
<th>Δ log r</th>
<th>Log r</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.004616</td>
<td>-5.003216</td>
<td>10.98759</td>
</tr>
<tr>
<td>Median</td>
<td>0.002091</td>
<td>-5.062976</td>
<td>7.861877</td>
</tr>
<tr>
<td>Max</td>
<td>1.003034</td>
<td>-3.024458</td>
<td>76.70115</td>
</tr>
<tr>
<td>Min</td>
<td>-1.362056</td>
<td>-6.360298</td>
<td>2.094463</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.207951</td>
<td>0.627335</td>
<td>11.24380</td>
</tr>
<tr>
<td>Symmetry</td>
<td>-0.746374</td>
<td>0.529160</td>
<td>3.793070</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>16.37818</td>
<td>4.125132</td>
<td>19.38627</td>
</tr>
<tr>
<td>Observations</td>
<td>172</td>
<td>173</td>
<td>173</td>
</tr>
</tbody>
</table>

Interest rate behavior has been modeled using the change in the log of up-to-59-days time deposit interest rate as the explained variable. Graph 5 depicts the evolution of the log.

Some empirical findings from the series seem to hold in general:

(i) The crisis as from late 2001 is clearly evident and characterized by a very high and persistent volatility.

(ii) The existence of (positive) jumps in the path is evident, even with logs and outside the period of crisis.

(iii) After the jumps, the interest rate tends to revert to an “equilibrium” level.

(iv) After the crisis, the interest rate also seems to show a trend to return to an equilibrium value from a lower level. It is not clear whether the “equilibrium” rate to which it is returning has changed.

(v) There is autocorrelation.
While later on we will provide arguments for the choice of the model for the interest rate path, empirical findings are already pointing towards making a distinction between periods and choosing a stochastic diffusion process complemented by a jump process.

**VI.1.b. The model for the short term interest rate**

We have looked for a model that, primarily, is tractable and, at the same time, yields a satisfactory statistical characterization of the empirical phenomena in the dynamics of the series, especially the trend to return to a long run average value and the presence of discontinuities.

The model explains the behavior of the monthly change of the log of the short term deposit interest rate. By using logarithms, the possibility of the nominal interest rate taking negative values is ruled out. Additionally, the specification of the equation to be estimated implies a mean reversion process, which makes the interest rate tend to return to a long term value after a shock. Two types of jumps can be identified: (i) jumps due to short term “surprises” linked to financial market turbulences, whose trigger is usually located abroad and (ii) shocks linked to mounting uncertainty at the end of 2001, the introduction of the “corralito”\(^{15}\) and the emergency period until mid 2003. The equation is as follows:

\[
\Delta \log r_t = c + a \log r_{t-1} + \sum b_i \Delta \log r_{t-i} + \sum c_i D_i^1 + \sum d_i D_i^2 + eDm
\]

\((6)\)

\(r_t\) stands for the short term deposit interest rate and \(\Delta\) is its first difference. Explanatory variables that turned out to be significant are: the explained variable lagged one and six periods, a constant and the log of the rate of the previous month. A set of dummy variables that control for exogenous jumps in the series (represented as \(D_i^1\)) also resulted significant. They capture jumps of the first type, that is, the Tequila crisis, Asia crisis, Russia crisis, the change of national president in Argentina in 1999 and the “blindaje” (shield)\(^{16}\) at late 2000. Another set of dummy variables (\(D_i^2\)) captures shocks of the second type. A further dummy variable (\(Dm\)) is also significant, which captures the fact that the interest rate has shown a steady tendency to grow as from April 2005, following a marked drop after the crisis (reaching very low levels in historical terms) and an immediately subsequent period in which it went alternatively up and down (Graph 6). ARMA terms for residuals did not turn out significant.

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\(^{15}\) Limitations on cash withdrawals from bank deposits under measures that became known as the “corralito”.

\(^{16}\) Substantial foreign financial help aimed at reducing uncertainty about the Government debt, at the end of 2000.
Academic models for interest rates usually show a higher sophistication compared with this model. On the other hand, however, this model yields a statistical characterization of empirical phenomena that is very satisfactory, which is difficult to find in the literature. We have left for an annex (Annex 4) a summary of the very vast literature on modeling the dynamics of interest rates, which also reflects the lack of consensus about the right specification of the underlying process. Academic models have mainly focused on pricing derivative contracts, on measuring interest rate risk arising from portfolios including these contracts and on modeling the term structure of yields. The short term interest rate gets special attention because it impacts on the whole term structure. Empirical studies have shown that the most traditional academic models (single factor diffusion models) would not adequately explain the dynamics of interest rates, leading to proposals of new formulations, such as GARCH, regime switching and jump models. These developments move along looking for an equilibrium between the level of difficulty of the models and their empirical adjustment.

Within the literature, the “reference” model in Pfann et al. (1996) has similarities with the autoregressive part of this paper’s model. The former is a linear AR(2) model of the changes in US Treasury bill return rates. The authors comment that the model suffers from heteroskedasticity and autocorrelation, and they go on to propose a threshold model. In our case, those specification problems would not exist, as is shown in the tests in Annex 2 (where we also show stationarity tests and econometrical results).

These results together with the familiarity and extension of the use of ARIMA models contribute towards proposing this type of models for Argentina. For example, some computer solutions permit the automatic estimation of this type of models. In our case, total automatization would not be possible because there is a need to identify the jumps; but the estimating process can be simplified to a large extent17.

The second part of the estimated model, which consists of dummy variables, allows the construction of paths that include a term of stochastic jumps. The model with jumps yields a statistical characterization of the behavior of the short term interest rate that is better than that which would be obtained from more complex diffusion or autoregressive models, as has also been found in some studies on short terms interest rates in the US18.

As for mean reversion, the model joins the group of models that show non-linearity of the trend of interest rates19.

V.1.c. Projecting interest rate paths in the simulation

In projecting the paths of the short term interest rate we do not rule out the possibility of experiencing the less disruptive type of shocks (type 1). On the other hand, we deliberately rule out the possibility of a 2001-2002 type of crisis taking place, considering the time horizon of the projection and the extreme and exceptional features of that crisis. Modeling such an event would be highly subjective and should be treated as a stress scenario, rather than within a VaR, or should be factored in within an exercise with a longer horizon. If that was the case, the transition probability from one regime to the other should also be modeled. The model for the high volatility regime would probably be different as well.

The equation used for the projections is, then, the following:

---

17 In general, modeling alternatives require numerical methods.
18 Das (2002)
\[ \Delta \log r_t = c + a \log r_{t-1} + \sum h_i \Delta \log r_{t-i} + eDm + \sigma dz_t + Jd\pi(h) \] (7)

The dummy variable \( Dm \) is set to 1, as it indicates a different tendency after the crisis, \( \sigma dz \) is the diffusion process, or random term for “normal times”, where the volatility \( \sigma \) is that of the residuals from Equation 6 and \( dz \) is a white noise innovation. The last term is a process that introduces random jumps \( J \) due to shocks. The occurrence of jumps is governed by a \( \pi \) Poisson process, with frequency parameter \( h \). This has been calibrated by measuring the historical frequency of jumps (3% of the months). The size of the jumps \( J \) are obtained from a probability distribution which is calibrated using the distribution of the dummy coefficients obtained in the regression. These do not reject the Normal distribution hypothesis. The processes for the diffusion, the Poisson variable and the size of the jumps are independent.

Each month, we need to determine whether the path is affected by only the “normal” random term or also by the “shock” one. To do this, a uniform random variable is generated. If it adopts a value higher than 3% (i.e. the \( h \) parameter in the Poisson distribution), only the random term for normal times applies. This term result from multiplying a randomly generated variable from a \( N(0,1) \) by the standard error of the regression. If the uniform variable turns out to be less than 3%, the high volatility random term is also considered. The size of the jump is obtained randomly from a \( N(\mu,\sigma) \) distribution, where the mean and dispersion have been calibrated based on the coefficients of the dummy variables in equation 120.

Graph 7 shows some paths generated for the simulation and Graph 8 shows the distribution of projected interest rates for 6, 12, 18 and 24 months in the future.

Graph 7 / Examples of paths generated by the stochastic model for short term deposit interest rates (the simulation works with several thousand paths, only 2 are shown here)

---

An alternative way to understand the procedure is as a Bernoulli approximation. It is assumed that, in every month, there is either one jump or none. A Bernoulli approximation is obtained by defining an indicator variable \( Y=1 \) if there is a jump and \( Y=0 \) if there is none. A Bernoulli variable takes the value 1 with probability \( p \) and value 0 with probability \( q=1-p \). A Bernoulli process in the limit follows a Poisson distribution with frequency parameter \( h=p \).
VI.1.b. The model for the consumer inflation rate

In turn, the consumer inflation rate model does not use logarithms given that it may adopt negative values, and the first difference is not applied given that the series is stationary in levels (see Annex 3). The model’s specification follows the same premises as those for the deposit interest rate model. In contrast to that model, there are no extraordinary jumps in months outside the period of the 2001/2002 crisis.

\[
\text{inf}_t = c + \sum a_i \text{inf}_{t-i} + \sum b_i D_i + d Dm + \sum e_i ar(i) + \sum f_i ma(i)
\]  

(8)

The variable \(\text{inf}_t\) represents the twelve month accumulated inflation rate, as measured in month \(t\). Significant explanatory variables are: the explained variable lagged one and two months, some dummy variables for months linked to the crisis \((D)\), one dummy variable for the post-crisis period \((Dm)\), an autoregressive term for six month lagged errors and a twelve month
moving average also of errors. The constant $c$ is not significant but it is nevertheless included given its economic meaning (it allows the estimation of a central trend value).

An accumulated inflation variable has been chosen because the model should be appropriate in its monthly behavior as much as in the accumulated inflation that it generates. In spite of the observed high variability of the monthly inflation rate, accumulated inflation shows a much less volatile evolution. In projecting a monthly path of annual inflation rates, a series of one month inflation rates is implicit. Annex 3 shows stationarity tests, estimated model coefficients and descriptive characteristics of the residuals of the regression.

In order to project, the dummy variables indicating the crisis are disregarded and the dummy variable that indicates the period after the crisis is set at 1. Initial values for the simulation, (inflation rate and interest rate) are those of December 2006.

Graphs 9 and 10 show the historical series and some projected paths, respectively. Graph 11 illustrates the distribution of projected annual inflation for 6, 12, 18 and 24 months in the future.
VI.1.e. Correlations

No significant correlations could be found between inflation and interest rates with different lags, nor between the errors of the corresponding regressions. Therefore, paths are projected taking both variables as independent. While it is unquestionable that, given a very strong and steady increase in any of the two variables it is expected that the other will follow suit, even if only partially, in our projections this kind of movement is not generated because extreme crisis scenarios are not included. Time deposit interest rates can undergo exogenous shocks of considerable magnitude but, following historical evidence, these shocks tend to revert relatively fast and do not transfer notably into inflation.

Simulation Results

Estimated models are frequently adjusted to be risk neutral, in the literature that, for example, prices derivatives. However, this is not necessary for other uses and, instead, a real (not risk
neutral) process must be used; for example, in risk management and in particular when running simulations where the objective is the estimation of a VaR measure, such as the case under study, or in cases of real options and first hitting time simulations.

The simulation consists in generating 5,000 paths (runs) for the risk factors and applying them to the payment structure of the benchmark portfolio. The contract’s economic value is calculated in each run (present value of future cash-flows, using the interest rate path generated in the simulation for discounting). A probability distribution of the contract’s economic value in the time horizon of the VaR (3 months) is thus obtained. VaR is then the quantile corresponding to the confidence level.

Table 3 / Real interest rate risk, nominal loan and adjusting by CER

Monte Carlo simulation (3 month VaR)

<table>
<thead>
<tr>
<th>4 years maturity</th>
<th>Fix interest rate</th>
<th>Interest rate + CER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19%</td>
<td>17%</td>
</tr>
<tr>
<td>Average PV</td>
<td>116.4</td>
<td>113.6</td>
</tr>
<tr>
<td>VaR 95%</td>
<td>8.0%</td>
<td>8.5%</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>13.1%</td>
<td>13.7%</td>
</tr>
</tbody>
</table>

The table shows the most important results for CER adjusting contracts (with different interest rates) and for fixed rate non-adjusting contracts. Take for example the column “4% interest rate + CER”. An Arg$100 loan with 4 year maturity, identical monthly amortizations, principal adjusting by CER and 4% interest rate, has an expected present value of Arg.$111.9. In the worst 5% expectable scenarios, the loan would produce a loss (measured as the difference with the expected value) exceeding 10.2% of that expected value, and in the worst 1% cases that loss would exceed 16.3%.

It can be seen that adjusting principal by CER produces an increase in risk: while 99% VaR of a fixed rate loan is around 13.5%, it increases to around 16.3% for an adjustable loan. This fact derives from the independence of inflation and interest rates: although the creditor is long in one of the risk factors (inflation) and short in the other (interest rate), the lack of a joint movement between these variables precludes a reduction in risk. On the contrary, while inflation dependence may be profitable for the creditor under some scenarios, it may well be to his detriment under others, and this fact is captured by VaR.

Note also that in order to yield a similar expected rate of return (i.e., the same average present value) the adjustable loan will have higher risk. This can be seen in the table comparing the two claims whose expected values are around Arg.$ 116 (CER adjustable + 6% interest versus nominal with 19% interest). This fact is in part a consequence of the different payment patterns of both types of claim.

The payment pattern of an inflation adjustable claim is different from that of a non-adjustable one (Graph 12). The adjustment of outstanding principal through time, following the evolution of CER, makes interest payments increase with that index, as they are determined as a fixed percentage on adjusted principal. Comparing payments from a nominal credit with those of an adjustable credit, both with the same maturity and amortization scheme, payments of the adjustable loan will be much more ahead in time. Using CER as discounting factor, principal and interest payments are fixed in real terms in the adjustable credit. That is why the IRR of an adjustable bond is usually quoted as a real rate of return (in terms of CER) and is calculated.
assuming no inflation as from the quotation date into the future. In contrast, coupon payments from a nominal claim decrease in time, in real terms.

**Graph 12 / Payment patterns of an inflation adjustable and a non-adjustable claim**

**VI.2.a. Outcome sensitivity to different features of the claim**

Different values of lending interest rates do not lead to significant changes in risk estimations. On the other hand, changes in maturity are important. For example, Table 4 shows, for different Modified Durations, the average 99% and 95% VaRs of loans that pay equal monthly amortizations and interest rates of between 2% and 6% on CER adjusting principal (using the Monte Carlo simulation approach).

**Table 4/ Real interest rate risk for different maturities, Monte Carlo simulation**

<table>
<thead>
<tr>
<th>MD</th>
<th>95% VaR</th>
<th>99% VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.0%</td>
<td>4.7%</td>
</tr>
<tr>
<td>2</td>
<td>16.3</td>
<td>10.0</td>
</tr>
<tr>
<td>4</td>
<td>26.8</td>
<td>17.7</td>
</tr>
</tbody>
</table>

*average for CER adjusting loans with different lending interest rates

The dynamic behavior of the variables, in particular the tendency to revert to an equilibrium level and the speed at which they revert, leads to measures of risk that are not proportional to MD. It can be seen that risk increases more than proportionally with MD when the latter changes from 1 to 2, but less than proportionally when MD goes from 2 to 4.

The simulation approach also has the advantage of being easily applicable to cash flows with irregular patterns, such as cash flows arising from the unmatched asset and liability portfolios of banks. Just as generated paths were applied to the structure of a simple claim (in this study), they could be applied to any portfolio mismatch scheme. The Delta Normal approach, instead, would produce the same outcomes for portfolios with the same MD, regardless of the timing of the gaps.
VIII. Risk adjusted return and return on capital, impact on lending rates

An analysis of rate of return on capital, assuming that capital equals risk (economic capital approach) and that the required return on it is an annual 15%, would indicate that, just out of real interest rate risk, a bank lending a CER adjusting loan like the benchmark would add around 250 basis points (bp) to its lending rate (16.3% of capital, 99% VaR, multiplied by 15%). On a fixed interest rate loan the bank would add up around 200 bp. For an 8-year loan, the amount of bp would be around 400.

But effects are different if capital equals regulatory required capital, because capital requirements according to BCRA regulations are more demanding on fixed rate loans than on adjusting loans. In fact, the BCRA’s capital requirement on fixed rate assets mismatches can be approximated as MD times 10% (derived from the published volatility). For our benchmark loan, this would produce a requirement of around 20%, while the simulation exercise indicates an economic capital of around 13.5%. Applying the same 15% target return on capital, a bank would compute 300 bp in its lending rate according to regulatory capital, instead of 200 bp that would arise from economic capital. On the other hand, the opposite takes place in the case of adjusting loans. The BCRA’s regulation does not factor in MD and requires approximately 3% of the asset’s value, while the risk indicated by the simulation exercise is slightly above 16%. Therefore, a bank targeting a 15% return on capital, would charge 45bp to its lending rate, instead of almost 250. This is a potentially distorting factor in banks’ risk/return allocation as CER mismatches are subsidized in regulatory terms in comparison to nominal interest rate mismatches21.

VIII. Risk premiums and the lack of indexation puzzle

Among asset pricing models, some argue (CAPM is one)22 that assets’ rate of return can be explained as the risk free interest rate plus a risk premium (or discount) in the case of those securities whose return is positively (negatively) correlated to the market portfolio return. This correlation represents the exposure to risk that cannot be eliminated by means of diversification.

It is quite clear that the Argentine banking system situation largely arose from a series of extraordinary events and that there is a lack of market alternatives to manage positions satisfactorily. Consequently, it is plausible that investors who are positively exposed to CER may require an additional return on these positions. Banks granting new loans that will show positive correlation with the return of the assets they already have in their portfolio (and which are strongly immobilized by the existence of regulatory benefits) would charge an additional margin to compensate for increasing present exposure. Thus, an additional risk premium should be taken into account when estimating the margin that banks could be adding on a CER adjusting loan. A model aiming at measuring this premium would demand data on comparable securities, adjusting and nominal, and the modeling of other factors that may be affecting pricing. This paper does not cover that quantification.

VIII.1. The outcomes and the lack of indexation puzzle

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21 The fact that CER mismatches have originated mainly connected to exceptional circumstances, especially public debt swaps, could be an explanation for the lack of a correction of the distortion until now, given that some could argue that such a correction would mean a change to the conditions that were valid at the moment of the swap, in addition to the high impact in terms of required capital.

A relationship can be found between the outcomes mentioned above and the seeming puzzle posed by the scarce development of indexed bonds markets\textsuperscript{23}.

It is usually argued that economic agents benefit from the existence of indexed securities given that these allow the reduction of risks associated with inflation. By buying indexed assets, an investor could lock in a real return until their maturity and, in that way, be protected against the possibility of an unexpected price increase eroding the real return of her investment. In turn, the issuer of indexed bonds would benefit because there would be a lower required rate of return on her debt, as a consequence of the smaller risk\textsuperscript{24}. This line of reasoning has led to the emergence of opinions and research papers supporting the indexation of contracts in the economy. However, as a matter of fact, there has been only a relatively scarce development of indexation, except for cases in which high rates of inflation “forced” its introduction, which would point in the direction of a seeming preference on the part of the agents to specify their obligations and claims in nominal units, in spite of the risk arising from unexpected changes in inflation. This contradiction led to the emergence, at a later date, of research papers that try to explain the seeming paradox of this scarce development\textsuperscript{25}.

This does not imply that this state of things is considered efficient. Some analyses on the topic point out that a broader use of indexation could be achieved together with a Paretian improvement in the economy welfare if a larger variety of indices existed—more particular indices, with less components.

The main idea behind the seeming puzzle (and our outcomes may provide a good illustration in this sense) is that indexation does not remove price risks but it substitutes relative price risk for price level risk. An indexed bond provides protection against the evolution of an aggregated level of prices, which is based on a specific basket of goods and services. If the agent buying that exposure does not consume that basket, she will be exposed to the risk of changes in relative prices. In the case of this paper, the bank “consumes” deposits to grant loans. If the claim provides the bank with a real return—in terms of a typical consumer basket— but its funding costs do not follow that basket, not only does the risk persist for the bank but it is also increased.

However, it is important to underline, in the framework of this paper, that this outcome derives from the fact of having ruled out the possibility of crisis scenarios in the simulation. Admitted scenarios are characterized by no correlation between inflation and time deposit interest rates. But historical experience shows that when inflation rockets to very high levels, time deposit rates tend to follow inflation. If the possibility of generating that kind of scenario was admitted, the difference of risk in favor of nominal claims would start to decrease, given that relative price risk would carry a lower weight in total risk and price level risk would carry a higher weight. This outcome could be obtained either if agents allocated some non-negligible probability to such scenarios or, also, if agents had a substantial ignorance or ambiguity regarding the probability distribution of future events. In other words, the outcomes above are valid for an interval of expected inflation rate, whose upper limit has not been determined but which is implicit in the model, when certain types of shocks are allowed and others are not.

When the probability of an extreme crisis is not nil or the uncertainty is very high, the indexed security will provide protection against extreme scenarios to the indexation buyer, but would also mean a higher risk to the indexed liability obligor. One can then wonder if there will be a

\textsuperscript{23} See Shiller et al. (1997) and Mukerji et al. (2000).
\textsuperscript{24} See, for example, Foresi et al. (1997), Wrase (1997), Hein et al. (2003) and Sack et al. (2004).
\textsuperscript{25} For example Mukerji et al. (2000) find that there are certain conditions under which there would be no transaction of indexed bonds in any equilibrium, when economic agents are averse towards ambiguity.
range of prices that will satisfy both parts, within which the securities will be exchanged. The literature on ambiguity aversion shows, with academic models, that there are ranges of prices for obligors and creditors, at which no indexed bonds are held.

This line of reasoning also indicates that there may be “hysteresis” in indexed bond transactions. If such bonds exist in an economy, for example because there has been an event of very high inflation that led to their introduction, or because the indexation of certain contracts is established by law (though inflation may be moderate, such as the cases of the UK and Israel), transactions with indexed claims are encouraged by the fact that agents already have indexed assets and liabilities in their portfolios and thus changing their positions may be optimal decisions for them.

Another implication of this line of analysis refers to the observed preference to use foreign currency denominated contracts instead of indexing them by a broad price index. This phenomenon could be explained if uncertainty, or ambiguity, regarding the probability distribution of the foreign currency is less than that regarding inflation.

Credit risk should not be left unmentioned, though it is usually ignored or assumed independent from interest rate risk. Certainly, this would not be appropriate if the cost of a debt increases speedily.

X. Conclusions

Holding CER adjusting assets in a portfolio funded by liabilities that do not apply that type of adjustment creates a significant risk, as has been shown in the case in which funding is in the form of short term time deposits.

Trying to quantify that risk in Argentina poses difficulties due to the scarcity of relevant long historical series and, thus, results should be taken with caution. But, in spite of error margins around estimated values, this paper estimates an order of magnitude of the risk. The banking system has been coexisting with this risk with significant amounts of mismatching and, no doubt, has found it an obstacle when facing the generation of new long term credit.

Applying a Delta Normal approach with linear approximation or a Historical Simulation approach to estimate VaR has been discarded because these techniques would not capture the dynamic features of risk factor evolution. A Monte Carlo simulation approach has been applied to estimate VaR.

With this approach, estimated risk for the benchmark asset (characterized by a 4 year maturity and monthly payments of principal and interest), is approximately 16% of economic value, according to a 99% VaR and with a 3 months time horizon. This risk is a little higher than that corresponding to a similar non-adjusting asset. The occurrence of an extreme crisis has been ruled out within the horizon of the simulation.

This exercise could illustrate the seeming lack of interest in indexing debt, which in the literature is identified as a puzzle. Clearly, if an economic agent does not “consume” the goods in the basket on which the adjusting index is based, indexation can increase risk, instead of decreasing it. A similar risk affects debtors whose liabilities adjust by CER if their income is not subject to adjustment.

The outcomes can also help to explain the limited development of derivative markets on CER adjustment, and they do not inspire great optimism on their future development. This follows from the fact that the most important players in the market have in general the same sign of
mismatch. (they are bought in CER)\textsuperscript{26}. Clearly, economic agents whose perception of future evolution of rates differs from that of the market (implicit in prices) may be willing to take on exposure, but only at the cost of high risk speculative positions.

As banks have a high exposure in their portfolios and it can only be decreased progressively, risk leads to higher required yields. Banks may, to a certain extent, compensate for increases in funding rates by raising lending rates on new loans. However, this would mean that, because of existing mismatches in their outstanding portfolios, they should raise their lending rates given an increase in the real interest rate for two reasons: first, due to higher funding rates, with no increase in spread; second, in order to increase spreads and compensate for losses stemming from unmatched outstanding portfolios. This kind of behavior would be procyclical and undesirable. Clearly, greater stability and/or predictability of funding rates and especially of inflation rates and the development of derivative markets would allow a reduction in risk and would encourage lower lending rates and more long term credit.

To conclude, relative capital charges for interest rate risk should be studied in order to avoid possible unjustified distortions.

\textsuperscript{26} Pension fund liabilities are implicitly adjusted by CER, as it seems reasonable to assure beneficiaries a steady purchasing power as measured by the consumer price index. But these institutional investors have legal restrictions on taking positions in derivative markets, and, even if regulations were loosened, they might not take them on.
Annex 1: The Modified Duration of an inflation adjusting bond

The Yield to Maturity or Internal Rate of Return of a bond is the discount rate that makes the bond current price equal to the present value of future cash flows. If B is the price, N represents remaining years until maturity, C stands for coupon interest rate and M for amortization payment at maturity, then the IRR is the (nominal) rate r that solves the following equation.

$$ B = \sum_{k=1}^{N} \frac{C \cdot M}{(1 + r)^k} + \frac{M}{(1 + r)^N} $$ \hspace{1cm} (9)

If future inflation is known, the price of an inflation adjusting bond is determined by the sum of nominal future payments multiplied by the value of those payments, such as in the following formula:

$$ B_i = \sum_{k=1}^{N} C \cdot M \cdot (P_{t+n} / P_t) \cdot \delta_i(k) + M \cdot (P_{t+n} / P_t) \cdot \delta_i(k) $$ \hspace{1cm} (10)

where \( \delta(k) \) is the discount function, expressing the value at \( t \) of a nominal payment that takes place periods in the future\(^{27} \).

When the bond principal is adjusted by inflation, “r” is a real interest rate and applying Fisher equation

$$ (1 + r)^N = \left[ \frac{(1 + i)}{(1 + \pi)} \right]^N $$ \hspace{1cm} (11)

the expression of the bond price is

$$ B = \sum_{k=1}^{N} \frac{C \cdot M \cdot (1 + \pi)^k}{(1 + r)^k} + \frac{M \cdot (1 + \pi)^N}{(1 + r)^N \cdot (1 + \pi)^N} $$ \hspace{1cm} (12)

where \( \pi \) stands for the inflation rate, which is assumed constant. Multiplicative terms indicating the inflation adjustment can be crossed out in all the terms. Thus, a expression equal to that of (9) is obtained, only that now principal M and discount rate r are expressed in real terms. When the semi-elasticity with respect to the discount rate is applied, to obtain the Modified Duration, the outcome is the same as in the case of a nominal bond, only that the discount rate with respect to which the sensitivity is measured is now a real rate.

\(^{27} \) For simplicity, the index equals one at moment \( t \), and the payment of the next coupon takes place in exactly one period.
Annex 2: The behavior model for time deposit interest rates

Null Hypothesis: \( \log \text{PFM} \) has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=13)

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<tr>
<th>t-Statistic</th>
<th>Prob.*</th>
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<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-3.106151</td>
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Test critical values:
- 1% level: -3.470679
- 5% level: -2.879155
- 10% level: -2.576241


Augmented Dickey-Fuller Test Equation
Dependent Variable: \( D(\log \text{PFM}) \)
Method: Least Squares
Date: 01/16/07 Time: 15:57
Sample: 1993:02 2006:08
Included observations: 163

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<th>Variable</th>
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R-squared 0.131830
Adjusted R-squared 0.120978
S.E. of regression 0.197397
Sum squared resid 6.287058
Log likelihood 34.01647
Durbin-Watson stat 1.961687

Null Hypothesis: \( D(\log \text{PFM}) \) has a unit root
Exogenous: None
Lag Length: 0 (Automatic based on SIC, MAXLAG=13)

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Test critical values:
- 1% level: -2.579226
- 5% level: -1.942793
- 10% level: -1.615408


Augmented Dickey-Fuller Test Equation
Dependent Variable: \( D(\log \text{PFM},2) \)
Method: Least Squares
Date: 01/16/07 Time: 15:59
Sample: 1993:02 2006:08
Included observations: 163

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R-squared 0.360589
Adjusted R-squared 0.360589
S.E. of regression 0.202893
Sum squared resid 6.668811
Log likelihood 29.21217
Durbin-Watson stat 1.944095

The significativeness of a constant and a trend were tested in the first place, using the log of the time deposit interest rate and its first difference. In the first case, a constant turned out to be significative, thus the test in this page shows a constant. In the second case neither the constant nor the trend resulted significant in the ADF test equation.
Dependent Variable: D(LOG_PFM)
Method: Least Squares
Sample: 1993:02 2006:08
Included observations: 163

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R-squared 0.864360  Mean dependent var -0.006165
Adjusted R-squared 0.843045  S.D. dependent var 0.211429
S.E. of regression 0.083763  Akaike info criterion -1.991552
Sum squared resid 0.982271  Schwarz criterion -1.555011
Log likelihood 185.3115  F-statistic 40.55187
Durbin-Watson stat 1.998601  Prob(F-statistic) 0.000000

Series: Residuals
Sample 1993:02 2006:08
Observations 163

Mean -1.91E-18
Median 5.42E-17
Maximum 0.168667
Minimum -0.244227
Std. Dev. 0.077868
Skewness -0.386142
Kurtosis 3.515953
Jarque-Bera 5.858703
Probability 0.053432
Sample: 1993:02 2006:08
Included observations: 163

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Null Hypothesis: INF_ACUM12 has a unit root
Exogenous: None
Lag Length: 2 (Automatic based on SIC, MAXLAG=13)

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The significativeness of a constant and a trend were tested in the first place and none of them turned out to be significant in the ADF test equation.
### Q-statistic

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### Series: Residuals

Sample 1994:01 2006:08
Observations 152

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Annex 4: Modeling the behavior of the short term interest rate. Review of literature

The risk-free interest rate has a fundamental role in finance. Academic models have aimed mainly at pricing and measuring the risk of contracts whose value is sensitive to interest rates. Especial attention is paid to modeling short term interest rates because they impact on the whole interest rate term structure.

There is a vast set of literature on modeling the dynamics of interest rates, particularly the short term rate, which reflects a lack of consensus on the right specification of the underlying process. These developments have evolved looking for an equilibrium between the level of complexity to solve the models and their adjustment to empirically observed processes.

In general, interest rates dynamics is modeled as process that has a deterministic and a stochastic component. In the most classic version, the first component is a trend that can incorporate a mean reversion process. The second component consists in a Gaussian term. In order to incorporate the heteroskedasticity observed empirically, in later works the Gaussian term is multiplied by the short term interest rate raised to a certain power. Different values of the power lead to different models, among them, the famous models of Vasicek and Cox-Ingersoll-Ross (CIR). When the power is not restricted to a specific value, the outcome is the general model of Chan-Karolyi-Longstaff-Saunders (CKLS).

A large number of empirical papers have argued that the CKLS model, and its particular forms, would not properly explain the observed dynamics of interest rates. For example, these models ignore the possibility of non-linear trends.

GARCH type models have been put forward in order to capture behavior patterns of conditional volatility (serial correlation). However, in contrast to most financial time series, interest rates show conditional volatility behavior patterns that not only are a function of past shocks on the rate but also can be a function of lagged levels of the same series. Recently, some papers have tried to combine single factor models with GARCH models.

Some researchers have extended the CKLS model adding a second and sometimes a third stochastic factor. Multi-factor models which capture short term rates empirical properties better than single factor models have also been used, but this is achieved at the cost of introducing more complexity and latent variables.

A further way to improve the empirical adjustment of single factor models, without introducing other factors, is by using threshold models, thus avoiding the most important numerical difficulties. Threshold models can capture changes in short term interest rate volatility that apparently depend on the level of that rate. An alternative is to expand the models with jumps. The introduction of jumps in the models decreases the statistical importance of a non-linear mean reversion. A number of papers differ in the assumptions on jump distributions.

More technical details are provided in what follows.

Single factor models

In spite of being too simple to model the complex dynamics of interest rates, single factor models are widely used in practice given their low level of difficulty. The most general model
among this type is that proposed by Chan, Karolyi, Longstaff and Saunders (1992), usually known as CKLS, which is given by

\[ r_t = \alpha + (1 + \beta) r_{t-1} + \epsilon_t, \]

\[ E(\epsilon_t) = 0 \]

\[ Var(\epsilon_t) = \sigma^2 r_{t-1}^{2 \gamma}. \]

or

\[ dr_t = (\alpha + \beta r_t) dt + \sigma r_t^\gamma dW_t, \]  \hspace{1cm} (14)

where \( dW_t \) is a Wiener increment.

This equation is also found in the literature in the following fashion:

\[ dr_t = \kappa (\theta - r_t) dt + \sigma r_t^\gamma dW_t, \]  \hspace{1cm} (15)

This model provides a simple description of the stochastic nature of interest rates, which is consistent with the empirical observation that interest rates tend to revert to an equilibrium value. The parameter \( \kappa \) (which is equal to \(-\beta\) from equation 14) determines the speed of mean reversion and \( \theta \) (which is equal to \(-\alpha/\beta\) from equation 14) is the level of central tendency. The variance of the process is proportional to the level of interest rates. The parameter \( \gamma \) denotes the volatility elasticity with respect to the level of interest rates.

A number of models, extensively known, are particular cases of this model. If \( \gamma = 0 \), the Ornstein-Uhlenbeck process is obtained, which is usually recognized by Vasicek (1977) work and denoted by the following specification:

\[ dr_t = (\alpha + \beta r_t) dt + \sigma r_t dW_t, \]  \hspace{1cm} (16)

One of the major criticisms to Vasicek model is that interest rates can assume negative values (even when this has a relatively low probability when time horizons are short and the process is calibrated to developed economies). The CIR model (Cox-Ingersoll-Ross, 1985), is obtained setting \( \gamma = 1/2 \) in the CKLS model. It has the advantage of assuring that the process is always positive if the parameters are within certain ranges. Vasicek model is also criticized because interest rate volatility is constant, while it is a stylized fact that volatility is lower when rates are lower. The CIR model also overcomes this criticism, given that volatility equals \( \sigma r_t ^{0.5} \).

Though CIR specification is better than Vasicek in explaining the volatility process, empirical adjustment is not very good.

Chan et al. (1992) compare eight different models, all of them obtained by modifying the value of the parameters in the general CKLS model. Their conclusion is that the dynamics of short term interest rates is better captured when the volatility of interest rate changes is highly sensitive to the rate level (high \( \gamma \)). Many papers have studied this topic in the United States (Treasury note yields). It has been observed that in some cases the processes may turn out to be non-stationary and that there are additional problems stemming from the incorrect specification of the conditional variance (its serial correlation is not captured).

Given the poor empirical results of the simplest versions of these models, more flexible models have been proposed, with non-linear trends and more general forms of the diffusion coefficient. For example, Aït-Sahalia (1996) proposes the following process:
GARCH models have been developed in order to capture volatility “clusters” (high volatility periods and low volatility periods). They have been extensively studied for exchange rates, equity prices and interest rates. Papers on this line of research can be found in Ghysels et al. (1996) and Shepard (1996). GARCH models present a problem in the case of interest rates given by the fact that rate volatility varies as a function of interest rate levels as well as as a function of squared innovations (basic structure of GARCH models). As a consequence of this specification problem, non-stationary GARCH models are obtained frequently.

Approaches combining both types of models (single factor and GARCH models) have been studied in recent years. Examples are Bali (2003), Brenner et al. (1996), Koedijk et al. (1997) and Longstaff and Shwartz (1992).

Having extensively studied single factor models and, apparently, having exhausted their ability to explain the short term interest rate process, academic developments proceeded in several directions. One of them has consisted in introducing other factors while another has allowed “regime switching” and/or jumps. For example, Jegadeesh and Pennacchi (1996) increase the CKLS model with a stochastic mean reversion trend. Ball and Torous (1999) and Andersen and Lund (1997) use a stochastic volatility as the second stochastic factor. However, these variations still find difficulties in replicating non-Gaussian innovations, or the thick tails of empirical distributions.

Models allowing regime switching may use Markov processes, such as in Gray (1996), Hamilton (1988), and Pai et al. (1998), among others. This approach captures some aspects of interest rate dynamics but is subject to two criticisms: (i) a latent (non-observable) variable is used and (ii) the term structure of the latent variable is difficult to analyze. An alternative, also introducing regime switching, is to let it be determined by the current level of the short term rate. This leads to threshold models, in which Tong (1983 and 1990) pioneered, while Pfann et al. (1996) took the lead in applying them to interest rates. Various studies within this field differ in the way of determining threshold levels and their number. When the model has a single factor and the threshold level is endogenously determined, it is called SETAR.

A model with a single threshold \( u \) and a single factor for the short term interest rate would be defined by the following process:

\[
\begin{align*}
    dr_t &= \left\{ \begin{array}{ll}
    (\alpha_1 + \beta_1 r_t)dt + \sigma_1 r_t^\gamma dW_t & \text{if } r_t \leq u \\
    (\alpha_2 + \beta_2 r_t)dt + \sigma_2 r_t^\gamma dW_t & \text{if } r_t > u
    \end{array} \right. 
\end{align*}
\]

This particular specification has only one threshold but there could be several. However, for the sake of parsimony, the number or regimes is typically low. Additionally, the threshold level could be determined by lagged variables or could be a function of the history of the short term interest rate. An alternative way to write the same dynamics is in the form of stochastic differential equations, using indicator functions:

\[
dr_t = 1_{[r_t \leq u]} \left[ (\alpha_1 + \beta_1 r_t)dt + \sigma_1 r_t^\gamma dW_t \right] + 1_{[r_t > u]} \left[ (\alpha_2 + \beta_2 r_t)dt + \sigma_2 r_t^\gamma dW_t \right]
\]
Switching regime models are not linear, but they keep the standard CKLS linear model in each stratum.

These models can generate the whole interest rate term structure via Monte Carlo simulations. However, it is necessary to know the market price of risk in order to do that. Parametrical techniques, which allow the estimation of the market price of risk with closed formulae for bond prices, have not been developed yet. In practice, these risk values are obtained by trial and error processes or by using non-parametrical methods. This approach has been studied in Aït-Sahalia (1992), Stanton (1997) and Cox et al. (1999), among others.

A mean reverting process of the CKLS type, complemented with a jump process would be:

$$dr = \kappa(\theta - r)dt + \sigma dz + Jd\pi(h),$$

(19)

where $\theta$ is the central trend parameter for interest rate $r$, which reverts at a speed $\kappa$. Thus, the interest rate evolves with a mean reversion trend and two stochastic terms, one is a diffusion and the other is a process producing stochastic jumps $J$. The occurrence of jumps is governed by a process $\pi$, which may be a Poisson process, whose event frequency parameter is $h$. The size of the jump $J$ can be constant or it can be obtained from a probability distribution. The diffusion, Poisson and jump-size processes may be independent.

When a jump process is added to the model, the non-linear trend becomes less significative. Ahn et al. (1988), Das (2002) and Johannes (2004) researched these models. In particular, Das incorporates jumps to the Vasicek model and finds strong evidence of jumps in the Federal Funds rate. Johannes uses a non-parametrical diffusion model to study Treasury bills rates in the secondary market and concludes that jumps are generally generated by macro-economic news announcements. A number of papers differ in the assumptions on jump distributions. Finally, there are studies which combine regime switching and jump models. Generally, it is concluded that the jump model should be conditional on the regime, given that discontinuous behavior predominates when the process goes through a high rate regime, while transitions are usually smooth within the more habitual low rate regime.

Models of interest rate term structure have been studied especially in relation to pricing securities whose payments are distributed along time. Literature is large and goes back to the beginnings of the XX century. The expectation hypothesis was the preeminent academic model before the development of the non-arbitrage approach (Cox et al., 1985). Basically, the hypothesis assumes that premiums on zero coupon risk free bonds are constant throughout time. The other early theories on term structure -liquidity preference and preferred habitat- may be seen as extensions of that hypothesis, adding predictions about premium size as a function of bond term. Generally, empirical data have not supported these hypotheses.

Thus, research about premium dynamics followed. One of the most important and foundational works is possibly the model with two estate variables by Brennan and Schwartz (1982). Most of these models do not allow a closed form solution for the term structure and must resort to numerical procedures, such as dynamic programming. They require measuring the market price of risk.

In order to overcome the difficulties posed by estimating the price of risk, two “preference free” approaches have emerged. One of these approaches uses lattice models, and includes those works by Ho and Lee (1986) and Black et al. (1990). The other approach employs the martingale concept and its main exponents are Hull and White (1990) and Heath, Jarrow and Morton (1990), or “HJM”.

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There is a group of models within the HJM framework, which explain the dynamics of instantaneous forward rates. A central idea to this framework is to recognize that there is an explicit relationship between forward rate trend and volatility parameters, which is derived from the condition of non-arbitrage opportunities. In differential notation:

\[ df(t,s) = \alpha(t,s)dt + \sigma(t,s)^T dW_t \]  

(20)

where \( W_t \) is a d-dimensional vector of risk neutral Brownian movement components, which determine the stochastic processes of the many \( f(t,s) \) forward rates, for \( s \in [T,t] \). Traditional models are contained within the HJM framework (when volatility \( \sigma(t,s) \) is assumed to be deterministic).

More recently, there has emerged an approach to term structure of interest rates which studies “affine” models. These models allow working out the whole interest rate term structure. Again, Vasicek and CIR models are contained in this group. Affine models can be extended to include jumps or to study the pricing of securities with contingent payments.

In a simple affine model, yields are “affine” functions of a estate variable vector \( x \) (i.e., they are related through a constant plus a linear term). The yields \( y^{(\tau)} \) of a bond with \( \tau \) periods to maturity are articulated as:

\[ y^{(\tau)} = A(\tau) + B(\tau)x \]  

(21)

for coefficients \( A(\tau) \) and \( B(\tau) \) depending on term \( \tau \). Restrictions on the functions \( A(\tau) \) y \( B(\tau) \) make yield equations consistent among them, for different terms. Models define the processes for estate variables \( x \), which can also be affine. See Das and Foresi (1996), Duffie, Pan and Singleton (2000) and Piazzesi (2005).
Bibliography


