Financialization of Commodity Markets: Non-linear Consequences from Heterogeneous Agent Behavior

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Abstract

Up to the financial slump of the second quarter of 2008 commodity prices grew fast for several consecutive years in a highly volatile context. Recent commodity fluctuations have raised both policy concerns and a prolific academic debate. This paper offers a coherent theoretical and empirical framework aimed at improving our knowledge of those elements driving commodity prices in the long run once the so-called process of “financialization of commodities” is incorporated into the analysis. To this end, we employ a smooth transition vector autoregressive model which is suitable for testing the hypothesis derived from a heterogeneous agent model in the commodity markets. The empirical methodology allows us to distinguish among those variables that influence prices in the long run –obtaining in this way an “equilibrium” or “fundamental” price; and the mechanisms that generate, strengthen and eventually correct short run deviations with respect to that equilibrium. The results suggest that high discrepancies between spot and fundamental prices tend to be corrected relatively fast, while small misalignments tend to persist over time without any endogenous correcting force taking place.

JEL Classification Numbers: C32, D84, Q11.

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**Introduction**

During the two decades following the oil shocks of the seventies the issue of commodity prices lost prominence in academic and policy debates. However, in the last five years we saw that nominal peaks in many commodities were accumulating in an impressive row. Records were broken in almost every month from the last part of 2007 and to the first half of 2008 in main commodity markets such as oil, copper, nickel, soybean or rice just to mention a few. But the rising trend in prices was spread out all over commodity varieties and hence, it results crucial now to understand the global causes and consequences of this sharp increase and its following violent reversion.

It is clear now that policymakers and academics did neither forecast the intensity nor the speed of recent commodity price movements. Therefore, the ongoing research agenda includes two problems: to understand what elements explain price movements; and to review policy responses in the light of this new scenario. In this paper we cover the first issue of the agenda, although we will point out the main policy challenges at the end of the document.

In explaining reasons behind price movements there are roughly speaking two stories not necessarily well connected, as Krugman (2008) has advocated.

The first story is basically about fundamentals. It says that world income is growing at a pace which is not matched by the supply side of the commodity markets. China and emerging Asia are the main characters here because living standards are increasing more than proportionally precisely in countries that have high income commodity demand elasticity due to Engel law (IMF, 2006; Kaplinsky, 2006; OECD-FAO, 2008). There is also room in this view for big dollar movements. The influence of this variable over commodity prices has been discussed in other historical booms and busts cycles (Ridler and Yandle, 1972; Dornbusch, 1985; or Borensztein and Reinhart, 1994). Also loose monetary conditions and excess of international liquidity are critical elements of this view, since they add inflationary pressure which tends to be reflected rapidly and with more intensity in auction markets like commodity ones (Frankel, 2006; Lipsky, 2008).
Finally, if the focus is posited on food commodities it will be necessary to add biofuels as a new determinant of prices (UNCTAD, 2006; IMF, 2007).

The second story points out speculation as the driving force of recent commodity ups and downs. It stresses the relevance of the so-called “financialization of commodities”, a process according to which a number of non-conventional actors such as investment banks, hedge-funds or pension funds have been investing in commodity-linked instruments.

Of course it could be asserted, as we do in this paper, that both stories overlap and are connected and that a complete picture should take all the pieces together in a coherent way. We argue that the impact of financialization and speculative activity is reflected on short run price dynamics, but not in the long term equilibrium. Specifically, we propose that financialization generates a non-linear adjustment pattern of commodity prices to its fundamental value.

Thus, in the theoretical front, we develop a simple heterogeneous agent based model in commodity markets that include chartists, fundamentalists and portfolio managers. An outcome of this framework is that price adjustment to equilibrium is reached in a non-linear way, being more intense as long as the past gap between the spot price and the equilibrium price increases.

Concerning this commodity price equilibrium, we assert it depends on determinants highlighted by previous literature: world demand, real exchange rate of the United States, real interest rates and the Prebisch-Singer hypothesis.

A novel characteristic of this paper is that it employs an empirical methodology that allows us to distinguish permanent and transitory movements in prices once the long run equilibrium is estimated. Thus, in modeling short run dynamics we make use of a smooth transition autoregressive model (STAR) which is suitable for testing the hypothesis derived from our heterogeneous agent based model in commodity markets.
The intuitive idea of the STAR model is that the discrepancy between current and fundamental prices plays a double role. On the one hand, it is the force that drives price changes in the required direction to fill the existing gap as in any traditional error correction model. On the other hand, the misalignment acts also as a transition variable, governing the state of the model. The larger the misalignment is the faster the speed of convergence will be.

The organization of the paper is as follows. In the next section the past track of commodity markets is described. The analysis suggests there are significant differences in the commodity outlook depending on both the time window employed and whether real or nominal indexes are considered. After that, we review theoretical and empirical literature of long run commodity price determinants. The issue of financialization is discussed in the third section. We conclude from it there is some evidence that at least does not contradict our hypothesis that financialization is important in influencing short run price dynamics rather than equilibrium levels. Following this, we present a stylized model of heterogeneous agents in commodity markets that gives support to our hypothesis. In the next section the econometric methodology is summarized. The empirical evidence is shown in the sixth part of the paper considering both long run and short run commodity behavior. Finally, the conclusions and policy challenges are presented.

1. Stylized facts of commodity prices

Up to the intensification of the financial slump during the second part of 2008, commodity prices grew in nominal terms at a strong pace for six consecutive years. However, the crisis has showed us again that commodity price flexibility is remarkable and those record values by June-July, 2008 were cut at least by a 30% at the end of October. In Figure 1 we have drawn the evolution of some key commodity prices from 2002 to 2008.
Contrary to the common belief that commodity prices have reached currently historical high levels, long-run perspective shows stagnant or decaying prices if we incorporate into the analysis the world inflation. In Figures 2 and 3 the long run trends of food, metals and oil in nominal and real terms are presented. The real series are deflated using US consumer price index (CPI).
When food and metals price indexes in real terms are analyzed, for example, we indeed verify between 2002 and 2008 an increase of 170% and 290%, respectively. Nevertheless, when the period 1960-2008 is considered, we observed that even after the last boom, real food prices are far below of their level in 1960.
(-41.84%) and real metals prices have just recovered the levels exhibited in that year. The oil story is quite different. Real oil price have risen strongly in the last decade, being its current level five times the figures it presented in 1960.

Some authors have stated that, instead of long-run trends, the most remarkable feature of commodity price dynamics is short and medium term volatility. According to Deaton (1999) “what commodity prices lack in trend, they make up for in variance”. Cashin and McDermott (2002) find that commodity price volatility has increased notably since Bretton Woods breakdown at the beginning of the seventies. Figures 4 and 5 depict the real price volatility measured by the standard deviation of the monthly real price changes using a rolling window of 12 months.

**Figure 4. Food and Metal Real Price Volatility**
We found that, in fact, average volatility in post Bretton Woods era has doubled respect to 1960-1972 period in the case of food, while it has risen 40% when metal price index is analyzed. Oil has exhibited a more abrupt price volatility increase. During the first phase the volatility measure averaged 0.0058 whereas in the second one that figure reached 0.069.

After summarizing stylized facts, we will try the rest of the paper to disentangle underlying factors that influence commodity prices in the long run – obtaining in this way an “equilibrium” or “fundamental” price; and the mechanisms that generate, strengthen and eventually correct short run deviations with respect to that equilibrium after Bretton Woods breakdown.

2. Long run drivers of commodity prices

In one of the most controversial thesis in the international economics field over the past century, Prebisch (1950) and Singer (1950) claimed that, contrary to the classical view, primary product prices would fall relatively to those of the industry. Prebisch (1950) asserted this tendency would be the outcome of a fundamental asymmetry in the international division of labor. Thus, while countries at the "center" had
kept all the gains of its productivity increases, "the periphery" had conceded parts of the benefits of its own technological progress.

The influence of this hypothesis over empirical research on commodity prices has been substantial and explains why the primary way of studying these prices has been through univariate methods such as unit root test or structural breaks tests.¹

A different approach for studying commodity prices starts asking which macroeconomic factors could have a clear connection or act as determinants of them.

In this sense, the pioneering model of Ridler and Yandle (1972) uses comparative static analysis in a single-good model to demonstrate that an increase in the real value of the dollar (i.e. a real exchange rate appreciation) should result in a fall in dollar commodity prices. Dornbusch (1985) constructs a simple supply-demand two country model to highlight this effect. In that paper, the elasticity of commodity prices to United States real exchange rate (RER) should conform to the following relationship:

\[
\frac{\partial \ln \left( \frac{P}{CPI} \right)}{\partial \ln \left( \frac{CPI}{eCPI^*} \right)} = -\frac{\beta^*}{(\beta \eta^* + \beta^*)}
\]

(1)

Where \( P \) is the price of a representative commodity basket; \( CPI \) and \( CPI^* \) are consumer price indexes in the United States and the rest of the world respectively; \( e \) is the multilateral nominal exchange rate (therefore \( \frac{CPI}{eCPI^*} \) is the RER); \( \eta \) and \( \eta^* \) are demand price elasticities of; and \( \beta \) and \( \beta^* \) are market shares of each country in the world demand. According to this theoretical model, this elasticity should lie

¹See for instance Grilli and Yang (1988), Cuddington and Urzúa (1989), Bleaney and Greenaway (1993), Cashing and McDermott (2002) or Ocampo and Parra (2003). The evidence that emerges from these papers is that negative growth rates tend to prevail when commodities are compared to industrial products considering the very long run. However, there is not a clear consensus about growth dynamics. While some authors have argued there is a decaying constant trend, other papers have stressed the importance of structural negative shifts that are not fully recovered during the upward phase of commodity prices cycles.
between 0 and -1. However, in empirical research it has usually been the case that it overshoots its theoretical value (i.e. is lower than -1).²

World demand is obviously another important driver of commodity prices. There is general consensus that Engel’s law is an accurate framework to predict the impact of income on food commodities (Houthakker, 1987; Hamilton, 2001). Hence, aggregate food income-elasticity in each country would fall as long as the transit to development is completed. In the case of metals, it has been argued there is an inverse U-shape relationship between its use and income level. Thus, the consumption intensity of metals increases up to a point in which GDP per capita reaches approximately 15,000 or 20,000 PPP adjusted USD (IMF, 2006) and then it starts to go down. In empirical models, indexes of world industrial production have been employed to measure world demand.

Apart from real exchange rate and industrial production, a third variable has been suggested as a determinant of commodity prices, namely the real interest rate.

Explaining the excess of co-movement of commodity prices with respect to fundamentals, Pindyck and Rotemberg (1987) consider that these movements are the result of herd behavior in financial markets since its participants could believe that all commodities tend to move together. The authors claim that, as storable assets, commodities are affected by expectations. Interest rate might affect the harvest or production in a number of commodities changing its future supply and so current prices. It could also affect expectations about future economic activity and then future commodity demands which, again, impacts on spot prices.

Frankel (2006) remarks that rising interest rates are transmitted to commodity prices through three channels: i) by increasing the incentive for extraction (or production) today rather than tomorrow; ii) by decreasing the desire of firms to carry inventories; and iii) by encouraging speculators to shift out of commodity contracts into treasury bills. The three channels of transmission work to reduce spot prices. In

fact, the author has argued that nominal records during 2006 in some commodities could be a signal that monetary policy has been too loose.

In our empirical non-linear model we will allow all these variables play an explicit role determining the “equilibrium” or “fundamental” long run price of a selected commodity basket.

3. Financialization of commodity markets

The debate regarding the issue of financialization in commodity markets has been intense and positions are diverse in this point. Some authors have blamed financial markets as the only element responsible for violent price ups and downs, while others have neglected their influence on prices. Hence, the key issue among financial market participants, academics and policymakers is trying to establish to what extent the financialization of commodities influence spot price levels and its stochastic properties.

According to their characteristics, the market for each commodity is divided into two parts. On the one hand, it is the physical spot market in which the consumers demand these goods to the producer and the spot price is determined. On the other hand, it is the financial derivative market where long and short traders agree on a future settled price. Derivative markets can be further decomposed in two categories: exchange markets where standardized contracts are traded through a central clearing entity, and over-the-counter markets (OTC) in which tailored contracts are negotiated, usually by means of a market maker.

With the term “financialization” of commodities the literature usually makes reference to two different though partially linked facts. The first fact is that derivative market activity has experienced an impressive growth in the last years. The second issue is the increase in the participation of financial investors in futures markets that occurred simultaneously.

Hence, one of the reasons to believe there is a close connection between price dynamics and speculative activity rests on the idea that we observed, during the past five years, a consistent rise in commodity prices in conjunction with a notable increase in turnover on commodity-linked instruments. In fact, trading volume in these instruments is several times higher than that of the physical production. Just to mention an
example, Domanski and Healt (2007) have pointed out that contracts in derivative commodity markets tripled between 2002 and 2005; while in the same period the ratio of financial activity of crude oil and copper to their world productions increased from 3.2 to 3.9 and from 30.5 to 36.1, respectively.

It is possible to illustrate this hypothesis comparing both the evolution of the number of outstanding commodity contracts and the amounts of USD outstanding OTC derivatives in conjunction with nominal commodity price movements. The exercise is presented in Figures 6 and 7 using the energy index and the non-fuel commodity index elaborated by the IMF.

**Figure 6. Evolution of derivative commodity contracts and commodity prices, 2002Q1 to 2008Q2.**

Source: IMF International Financial Statistics and BIS Quarterly Review
As it was mentioned before, there is evidence of a considerable rise in derivative market activity when the so-called “boom” cycle of commodity prices from the beginning of 2002 to the second quarter of 2008 is observed. During this period, total commodity contracts grew by 170%; while energy and non-energy commodity prices increased by 350% and 120% respectively. Moreover, the evolution of OTC derivatives in nominal USD amounts was even more impressive as Figure 7 reveals.

Thus, in the light of this evidence it results quite natural to associate this significant increase in financial commodity-market deepening with soaring prices.

However, there is an important caveat in this line of reasoning. Specifically, if financialization has actually played a fundamental role in boosting commodity prices, we would expect lower growth rates for those commodities that lack derivative markets. Next figures reproduce the exercises carried out by Deutsche Bank (2008) and Viñals (2008) consisting in calculating price increases in both exchange and non-exchange trade commodities.
In Figure 8, we observe that prices of exchange traded commodities have appreciated by a similar, if not lower, amount to non-exchange traded commodities where financial activity is not possible.

Viñals (2008) has performed a similar exercise considering just the final part of the commodity boom (Figure 9).
Again, we cannot establish a clear cut line between exchange and non-exchange traded commodities in terms of price variations. In some sense, this refutes the claim that commodity price increases that took place until the first half of 2008 were driven largely by speculative activity.

A more careful analysis of the issue of financialization requires going deep into the microstructure of derivative markets. Among of these market participants, there is a first wide division between commercial hedgers and financial investors.

Commercial hedgers are buyer or sellers of the physical commodity who use derivatives to hedge against the risk of price fluctuations. In the end, this type of agent is interested in the evolution of future spot prices of the underlying commodity (IMF, 2006).

Financial participants have different incentives from commercial hedgers. We can distinguish two strategies among them. On the one hand, there are “buy and hold” investors who pursue fully collateralized long-only future strategies, i.e., acquiring a long position in futures and investing the same amount in treasury bills as collateral. This strategy is usual among pension and mutual funds and it has
historically had excess returns similar to those of equities. Additional interesting properties of commodity futures as a buy and hold strategy is that their returns are negatively correlated with equity and bond returns, and they prove to be a good hedge against unexpected inflation (Gorton and Rouwenhorst, 2004; Erb and Harvey, 2005).

On the other hand, we have a broader group of investors pursuing more complex strategies. Hedge funds have recently played an active role. Their operations are characterized by freedom in using wide range of instruments, ability to short sell and high leverage (Stefanini, 2006). Also, retail investors are becoming increasingly important since they could participate in new instruments such as Exchange Trade Commodity Funds or Structured Commodity Notes (McNee, 2006; Bienkowski, 2007).

Although it could be argued that the action of financial investors have increased in the last years, this is not entirely a totally new phenomenon. This statement can be verified in Figures 10 and 11, where the number of open positions (long plus short) of financial participants in key future commodity markets since 1986 is depicted.

**Figure 10. Non-commercial open positions in oil, gold and copper, 1986M1, 2008M5**

Source: Calculations based on Deustche Bank data
We observe a strong increase in the number of contracts opened by financial investors in the oil, gold and copper markets since the beginning of 2002; but the activity previous to that date was far from negligible. The picture is quite different from soft agricultural commodity markets in which we observe a sharp drop in financial activity connected to the Asian crisis of 1998 (Figure 11). After that, the number of open contracts just recovered its previous levels. From this evidence we conclude that financial activity has intensified in the last five years but could not be viewed as a completely new fact.

Besides, the impact of speculative activity on prices could depend more on net positions (long minus short) rather than open positions. In Figure 12 we have drawn these net positions of financial participants in selected derivative markets and the corresponding spot price (expressed as an index).
Figure 12. Net positions of financial investors in selected commodity markets, 1986M1, 2008M5

Source: Authors calculations based on Deustche Bank and IMF data
There are several relevant points to highlight from this figure. Firstly, it seems that the mean position of financial investors throughout time tends to be a long one, but this fact could be partially explained by institutional investors going into long “buy and hold” strategies. There is so a positive “long bias” among non-commercial participants.

In the second place, we can observe that net financial positions tend to be volatile which means that non-commercial investors could either act as a push or pull factor of influence over prices depending on specific circumstances.

In the third place, it seems there is a positive correlation between the net position and the spot commodity price which indicates that high price levels induce appreciation expectations of financial participants.

Finally, with the notable exception of copper, it was the case that the last phase of high prices from 2005 went along with short net non-commercial positions. However, it is important to note that apart from the most recent case of copper, there were other phases in the past in which net long positions coexisted with stagnant commodity prices. In wheat for instance, we observed a 200% increase in prices between 2005 and 2008 which coincides with a slight long position during the whole period. Moreover, we could also note from Figure 12 that price drops were not necessarily followed by aggressive net short positions.

Explicitly, all this evidence means it is not necessary to have neither derivative markets nor aggressive financial investor participation to observe sharp price fluctuations.

In the financialization debate there is a last remark connected with the issue of causality. We think that a hypothesis that deserves some attention is that higher prices during the past five years could have caused an increased interest in commodity investment, and not the other way around as it is usually stated.

To shed some light about this hypothesis we have calculated in Table 1 the correlation coefficient between net financial positions and the growth rates in various commodity prices using different time windows. We take the net financial positions at the end of the month and compute firstly the contemporaneous correlation with the price variation taking place between this date and the previous month. Then we
consider both backward and forward price variation windows of up to three-months. In this way, the pairwise correlation between open positions and price changes at \( t + 3 \) considers the accumulated three month forward price variation. As a complementary exercise we have added the corresponding Granger causality test to establish if changes in financial positions anticipate spot price changes or if it is the other way around.

**Table 1. Net financial positions and commodity prices, correlations and causality, 1986-2008**

<table>
<thead>
<tr>
<th></th>
<th>COPPER</th>
<th>GOLD</th>
<th>SILVER</th>
<th>WTI</th>
<th>SUGAR</th>
<th>SOYA</th>
<th>MAIZE</th>
<th>WHEAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T \ vs \ T-3 )</td>
<td>0.35</td>
<td>0.64</td>
<td>0.53</td>
<td>0.39</td>
<td>0.09</td>
<td>0.52</td>
<td>0.58</td>
<td>0.34</td>
</tr>
<tr>
<td>( T \ vs \ T-2 )</td>
<td>0.37</td>
<td>0.59</td>
<td>0.50</td>
<td>0.39</td>
<td>0.07</td>
<td>0.51</td>
<td>0.56</td>
<td>0.37</td>
</tr>
<tr>
<td>( T \ vs \ T-1 )</td>
<td>0.38</td>
<td>0.50</td>
<td>0.46</td>
<td>0.40</td>
<td>0.05</td>
<td>0.47</td>
<td>0.50</td>
<td>0.39</td>
</tr>
<tr>
<td>( T \ vs \ T )</td>
<td>0.39</td>
<td>0.42</td>
<td>0.44</td>
<td>0.41</td>
<td>0.02</td>
<td>0.40</td>
<td>0.42</td>
<td>0.37</td>
</tr>
<tr>
<td>( T \ vs \ T+1 )</td>
<td>0.22</td>
<td>0.27</td>
<td>0.17</td>
<td>0.16</td>
<td>0.07</td>
<td>0.20</td>
<td>0.26</td>
<td>0.21</td>
</tr>
<tr>
<td>( T \ vs \ T+2 )</td>
<td>0.19</td>
<td>0.29</td>
<td>0.13</td>
<td>0.11</td>
<td>0.15</td>
<td>0.15</td>
<td>0.24</td>
<td>0.20</td>
</tr>
<tr>
<td>( T \ vs \ T+3 )</td>
<td>0.20</td>
<td>0.34</td>
<td>0.14</td>
<td>0.12</td>
<td>0.18</td>
<td>0.12</td>
<td>0.21</td>
<td>0.17</td>
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Granger Causality Test

<table>
<thead>
<tr>
<th></th>
<th>COPPER</th>
<th>GOLD</th>
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<th>SOYA</th>
<th>MAIZE</th>
<th>WHEAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Variations do not Granger Cause Changes in Net Financial Positions</td>
<td>1.77820**</td>
<td>0.67559</td>
<td>0.86989</td>
<td>0.80581</td>
<td>0.30402</td>
<td>0.98956</td>
<td>1.57599*</td>
<td>0.82225</td>
</tr>
<tr>
<td>Changes in Net Financial Positions do not Granger Cause Price Variations</td>
<td>0.76235</td>
<td>2.785***</td>
<td>1.05728</td>
<td>1.29875</td>
<td>0.80934</td>
<td>2.09115**</td>
<td>1.21476</td>
<td>1.56979*</td>
</tr>
</tbody>
</table>

Source: Authors calculations based on Deustche Bank and IMF data

It is clear that in all the cases there is a strong positive correlation of past price variation at time \( (t-1, \ t-2, \ t-3) \) that tend to lower in subsequent periods. This fact could indicate that financial speculators take long positions when they observe price increases in the recent past with the expectation of further increases in the future. In the next section, we will refer to this type of behavior as driven by simple “rules of thumb” or “chartist analysis”. The evidence of Granger causality tests, however, does not reveal any clear causality pattern. This means the issue of causality remains open and further research would be needed.

Taking together the pieces of information provided in this section we interpret this preliminary evidence as an indication that financial participants could induce price fluctuations or excess volatility in some markets, but do not have a long-lasting impact on equilibrium prices which are ultimately determined by
fundamental supply-demand factors. In the next section, we develop a simple model to stress a potential mechanism in which financialization alters the dynamic adjustment of prices to equilibrium. We will then test how actual commodity data fits with this hypothesis.

4. Heterogeneous agents in commodity markets

The model assumes that the change of the commodity price in the next period is determined by the interaction of three different agents\(^3\) called fundamentalists \((F)\), chartists \((C)\) and portfolio managers \((PM)\) in accordance to the following expression:

\[
\Delta P_{t+1} = a_1 E(\Delta P^C_{t+1}) + a_2 E(\Delta P^F_{t+1}) + a_3 E(\Delta P^{PM}_{t+1})
\]

Where \(E(\Delta P^C_{t+1})\), \(E(\Delta P^F_{t+1})\) and \(E(\Delta P^{PM}_{t+1})\) are the price change expectations of each agent and \(a_1, a_2, a_3\) are fixed weights that measure the relative importance of each group.

The expectations of the fundamentalists are based on the notion of commodity price reversion towards its long run equilibrium. The particular specification is:

\[
E(\Delta P^F_{t+1}) = -\alpha (P_t - F_t(X_t))\quad \alpha > 0
\]

Where \(F_t\) is the fundamental price of the commodity (or the relevant commodity price index) in time \(t\). This price is a function of a vector of variables \((X_t)\) stressed by empirical literature such as world demand for commodities, the real exchange rate of the United States and the real interest rate.

According to equation (3), fundamentalists expect decaying (increasing) prices when current prices are higher (lower) than fundamental prices. Thus, they are prone to sell or buy the commodity in a countercyclical fashion. It is not necessary to assume that fundamentalists know exactly which the long term value of the commodity is. We can think instead they can obtain a consistent estimation of this

\(^3\)Pioneering work on heterogeneous agents literature corresponds to Frankel and Froot (1987a,b and 1990), DeLong et al. (1990a,b), and Shleifer and Summers (1990).
equilibrium. For instance, it could be assumed that these agents have an imperfect knowledge about the real model because there exists uncertainty regarding the true value of the parameters, but they build their expectations based on an econometric regression without making systematic errors (Bray and Savin, 1986 and Fourgeaud et al., 1986).

Chartists, on the contrary, employ technical analysis and follow the current trends in prices. One way to formalize this type of strategy is:

\[ E(\Delta P_{t+1}^C) = \delta(P_t - P_{t-1}) \quad \delta > 0 \]  

(4)

Every time prices increase, these agents will take a long position in commodities because they expect that this trend will continue in the future. From the correlation analysis of the third section we know this is compatible with the actions of financial participants.

The key factor of this simple model is the inclusion of portfolio managers who are assumed to have an information advantage, in the sense they know the way the other market players form expectations. In order to take advantage of their knowledge, \( PM \) agents adjust their expectations employing a weighted average of expressions (3) and (4):

\[ E(\Delta P_{t+1}^{PM}) = (1 - w_t)E(\Delta P_{t+1}^C) + w_t E(\Delta P_{t+1}^F) \quad 0 \leq w_t \leq 1 \]  

(5)

It is crucial here the role of the variable \( w_t \) which governs the weight given to each type of expectation in time \( t \). We assume that \( w_t \) adjusts endogenously in response to the size of past misalignment. Thus, the \( w_t \) variable is the source of non-linearity into the model. We propose, in particular, the following exponential function:

\[ w_t = 1 - \exp\left(-\gamma(P_t - d - F_t - d(X_{t-d}))^2\right) \quad \gamma > 0, \quad d \geq 0 \]  

(6)

The intuition behind the specification of equation (6) is that the gap between actual and fundamental prices (lagged by \( d \) periods) is the element that determines the weights assigned to the expectations of \( F \).
and $C$ agents by the portfolio managers. When the case is such that $F_{t-d} \equiv P_{t-d}$, then $w_t$ will show a very small value encouraging portfolio managers to follow the behavior of chartists. In the limit, when $F_{t-d} = P_{t-d}$ ($w_t = 0$), the price change will be given by:

$$\Delta P_{t+1} = (a_1 + a_3)E(\Delta P_{t+1}^C) = (a_1 + a_3)\delta(\Delta P_t)$$

(7)

If we are interested in empirically studying commodity price dynamics, expression (7) suggests employing a purely autoregressive econometric specification.

As long as the gap between $F_{t-d}$ and $P_{t-d}$ increases, portfolio managers start to bet against this misalignment. That is, the larger the misalignment is, the larger the weight they give to $F$ expectations is. Again, in the extreme case $\exp(-\gamma(P_{t-d} - F_{t-d}(X_{t-d}))^2) \to 0$, and consequently $w_t = 1$ and $E(\Delta P_{t+1}^{PM}) = E(\Delta P_{t+1}^{F})$. After some substitutions, the law of motion for the price dynamics will be given by:

$$\Delta P_{t+1} = a_1\delta(\Delta P_t) - (a_2 + a_3)\alpha(M_t)$$

(8)

Where we have defined $M_t = P_t - F(X_t)$.

The parameter $\gamma$ is also relevant in this scheme, governing the speed in which portfolio managers adjust their expectations through the $w_t$ variable. If $\gamma$ is quite high, for instance, even a small misalignment will induce $PM$ agents to form expectations as the fundamentalists.

The general expression of commodity price changes can be obtained by replacing (3), (4), (5) and (6) into (2) and rearranging terms:

$$\Delta P_{t+1} = (a_1 + a_3)\delta\Delta P_t - a_2\alpha M_t - a_3(1 - \exp(-\gamma M_{t-d}^2))\Delta P_t - a_3\alpha(1 - \exp(-\gamma M_{t-d}^2))M_t$$

(9)

Thus, price dynamics depends on several factors. The first two terms are based on a standard error correction model: a purely autoregressive term and a linear error correction factor. The remaining terms are those which generate the non-linear adjustment pattern. In the empirical analysis the emphasis will be placed on the non-linear adjustment coefficient of the price deviation from the long-run equilibrium.
Summarizing, we have developed a theoretical model in which fundamentals continue to be the only real force that drives long run prices. However, heterogeneity in expectations among market participants is important in determining the adjustment properties to equilibrium.

It will be shown in the methodological section that a specific type of smooth transition autoregressive model (STAR) tracks the commodity price dynamics derived from the model and described by the equation (9). With this econometric specification it will be feasible to study both the fundamental equilibrium and the potentially non-linear adjustment properties of the implied misalignment.

### 5. Econometric methodology

In recent decades there has been a growing interest in the use of non-linear econometric techniques. Among them, models with switching regimes turn out to be particularly attractive because they incorporate a law of motion that governs the shift from one state to another. This law could be deterministic or stochastic. In the first case, the regime is determined by past values of observable variables, and it is known with certainty by all economic agents. In contrast, the state is stochastic if the regime is known only with some probability at every moment of time.

The simplest autoregressive model with deterministic regimes corresponds to the case of sudden changes and was developed by Tong (1978, 1990), Tsay (1989). In brief, it compares the transition variable ($T_V_t$) with a threshold ($c$) in order to split up the linear model into two sub-models. If an autoregressive specification without explanatory variables is assumed, then the threshold model will be:

$$y_t = \begin{cases} \phi_{1.0} + \phi_{1.1}y_{t-1} + \ldots + \phi_{1,p}y_{t-p} + \varepsilon_t & \forall y_t \leq c \\ \phi_{2.0} + \phi_{2.1}y_{t-1} + \ldots + \phi_{2,p}y_{t-p} + \varepsilon_t & \forall y_t > c \end{cases}$$  \tag{10}

In equation (10), the switch between states is determined by the comparison between the transition variable ($y_t$) and the threshold ($c$), and it occurs abruptly. The idea that the transition can be done gradually represents an important progress in this literature. It theoretically corresponds to the notion that
economic agents do not react simultaneously when new information is spread or when a shock hits the economy. This empirical strategy is also valid if the effects of structural changes materialize slowly.

Moreover, whenever economic intuition or theory suggests that a relationship among variables is valid under certain circumstances but is no longer true if these circumstances change, then a smooth transition autoregressive (STAR) model will be suitable for its empirical test. Chan and Tong (1986); Granger and Teräsvirta (1993); Teräsvirta (1994), or Franses and van Dijk (2000) are pioneering references of this approach.

The representation of a smooth transition autoregressive model of $p$ order or $\text{STAR}(p)$ is as follows:

$$
y_t = \left( \phi_{1,0} + \phi_{1,1} y_{t-1} + \ldots + \phi_{1,p} y_{t-p} \right) \left( 1 - F(TV_{t-d}; \gamma, c) \right) + \\
\left( \lambda_{2,0} + \lambda_{2,1} y_{t-1} + \ldots + \lambda_{2,p} y_{t-p} \right) F(TV_{t-d}; \gamma, c) + \epsilon_t
$$

(11)

Alternatively:

$$
y_t = \left( \phi_{1,0} + \phi_{1,1} y_{t-1} + \ldots + \phi_{1,p} y_{t-p} \right) + \\
\left( \lambda_{2,0} + \lambda_{2,1} y_{t-1} + \ldots + \lambda_{2,p} y_{t-p} \right) F(TV_{t-d}; \gamma, c) + \epsilon_t
$$

(12)

Where the following conditions are satisfied:

$$
\lambda_{2,0} = \phi_{2,0} - \phi_{1,0}, \quad \lambda_{2,1} = \phi_{2,1} - \phi_{1,1}, \ldots, \quad \lambda_{2,p} = \phi_{2,p} - \phi_{1,p}
$$

(13)

$$
E(\epsilon_t | \Omega_{t-1}) = 0
$$

(14)

$$
E\left( \epsilon_t^2 | \Omega_{t-1} \right) = \sigma^2
$$

(15)

$$
\Omega_{t-1} = \{y_{t-1}, \ldots, y_{t-p}\}
$$

(16)

In equations (11) and (12), the expression $F(TV_{t-d}; \gamma, c)$ is known as the transition function. It is a continuous function whose image is the interval $[0, 1]$.\(^4\) The $\gamma$ parameter measures how fast the

\(^4\) Both properties differentiate a smooth transition autoregressive model from a threshold model, because in the latter the transition function is direct, taking only two values: 0 or 1.
adjustment between regimes is, while the parameter $c$, establishes the limit point after which the switching between regimes start to take place. Finally, $TV_{t-d}$ refers to the transition variable with $d$ lags.

There are very few technical restrictions on the sort of variable(s) that could be $TV_{t-d}$. Usual options include lags of endogenous variables, exogenous variables, functions of endogenous and/or exogenous variables or a time trend (van Dijk et al., 2002).

On some occasions, however, a theoretical hypothesis suggests which variable can be regarded as a determinant of the transition. This is precisely the case of commodity prices and their possible pattern of non-linear equilibrium correction. From the heterogeneous agent model we obtain a specific theoretical restriction establishing that past misalignment between the current price and the fundamental value not only acts as a determinant of price changes but also as the variable that governs the transition between regimes.

Usual $TV_{t-d}$ functions are either logistic or exponential:

$$F(TV_{t-d} ; \gamma, c) = \frac{1}{1 + \exp(-\gamma(TV_{t-d} - c))}, \quad \gamma > 0$$

(17)

$$F(TV_{t-d} ; \gamma, c) = 1 - \exp(-\gamma(TV_{t-d} - c)^2), \quad \gamma > 0$$

(18)

The logistic function$^5$ allows us to distinguish between two regimes or states, named high and low regimes respectively. The high state arises from positive and large values of $(TV_{t-d} - c)$, since $\exp(-\gamma(TV_{t-d} - c)^2)$ tends to zero, and hence the expression (17) tends to 1. By contrast, the regime is low when $(TV_{t-d} - c)$ takes low values and so $\exp(-\gamma(TV_{t-d} - c)^2) \to \infty$ and $F(TV_{t-d} ; \gamma, c) \to 0$. The logistic specification is valid when it is believed that the transition takes place in a monotonic way.

---

$^5$A logistic smooth transition autoregressive (LSTAR) model is obtained when function (17) is applied.

$^6$Since the transition function can take any continuous values between zero and one, characterization of a STAR model with only two regimes may look quite arbitrary, particularly in those cases in which the smoothing parameter is low and, therefore, there are many intermediate values of the transition function. In this sense, van Dick et al. (2002) argue that the STAR model can be thought of as a methodology that allows a "continuous" set of regimes.
On the contrary, the exponential function is useful if the value of absolute deviation of the transition variable with respect to parameter \( c \) is the important feature. This specification is known as ESTAR model (exponential smooth transition autoregressive model) and restricts the dynamics of the equation to be the same alongside the extreme values of \( F(TV_{t-d}; \gamma, c) \).

Thus, in an ESTAR model the asymmetries between regimes are given by the absolute magnitude of the differences rather than by their sign.

Some well-known applications of STAR models could be found in papers studying misalignment of the real exchange rate regarding its fundamental value given by the purchasing power parity (Michael et al., 1997; Taylor et al., 2001, or Chen and Wu, 2000); in the literature of non-linear adjustment of deviations from uncovered interest rate parity (Sarno et al., 2006); or in those works that test non-linear mean reversion in stock futures (Monoyios and Sarno, 2002).

The extensions of STAR models to multivariate contexts (VAR models or systems of equations) have been studied, among others, by Weise (1999), van Dijk (2001), Camacho (2004) and Mendoza (2004).

The general structure of a non-linear equilibrium correction model is:

\[
\Delta Y_t = \Pi_{1,0} + \Gamma D_t + \alpha_1 M_{t-1} + \sum_{j=1}^{p} \Pi_{1,j} \Delta Y_{t-j} + \\
\left( \Pi_{2,0} + \alpha_2 M_{t-1} + \sum_{j=1}^{p} \Pi_{2,j} \Delta Y_{t-j} \right) F(TV_{t-d}; \gamma, c) + \epsilon_t
\]

(19)

Where \( \Delta Y_t \) is a (nx1) vector, \((\Delta y_1, \Delta y_2, ..., \Delta y_n)\), \( D_t \) is a (mx1) vector of dummies which control for outliers and \( \Gamma \) (mxn) is the respective matrix of coefficients. Also, \( \Pi_{1,0} \) and \( \Pi_{2,0} \) are (nx1) vectors containing the constants of the linear and non-linear part in each case. The \( \Pi_{1,j} \) and \( \Pi_{2,j} \) are (nxn) matrices for \( j : 1, ..., p \) that correspond to the autoregressive coefficients. The equilibrium correction term is denoted by \( M_t = \beta' X_t \), where \( \beta \) is the (nxr) matrix of coefficients of the long-term relationship and
\( X_t \) is a (nx1) vector which stands for the variables in levels. Finally, \( \alpha_1 \) and \( \alpha_2 \) are (nxr) matrices formed by the adjustment coefficients of deviations from long-term relationships, where \( r \) indicates the number of cointegration equations. These coefficients play a fundamental role in the model since they capture the linear and non-linear adjustment pattern.

Concerning the operational steps needed to implement a STAR model, Teräsvirta (1994) proposes a procedure for the univariate case, whereas Granger and Teräsvirta (1993) examine the multivariate case and Camacho (2004), among others, extends it to a multi-equational approach. In all cases, the process encompasses basically four stages: i) estimating a linear model ii) testing non-linearity; iii) estimating the non-linear model, and iv) computing non-linear impulse-response analysis.

The first step is to estimate a linear model that will serve as a benchmark to contrast the non-linearity hypothesis. The estimation sequence follows common techniques of time series analysis. In this point, it is important to control for outliers and to check the behavior of the residuals regarding autocorrelation and heteroscedasticity.

The second stage consists of a linearity test. If the alternative hypothesis to the linearity is the smooth transition exponential model (STAR)\(^7\), Teräsvirta (1994) suggests using a first order Taylor expansion \((T^\epsilon_t)^7\) to obtain an auxiliary regression which may serve as a base to contrast the null hypothesis. To this end, one should take equation (18) and calculate \( T^\epsilon_t \) in \( \gamma = 0 \). After some simplifications, the following auxiliary regression will be achieved (vector notation):

\[
Y_t = \beta_0 X_t + \beta_1^r X_t TV_{t-d} + \beta_2^r X_t TV_{t-d}^2 + \omega_t \quad (20)
\]

\(^7\)The auxiliary regression is modified when the specification is done throughout a logistic function. There is also evidence that expression (20) is appropriate when there is no knowledge (or no prior intuition) whether the relevant alternative is the expression (17) or (18). In this regard, see Luukkonen et al. (1998).
Where \( X_t = [y_{t-1}, y_{t-2}, \ldots, y_{t-d}] \); \( \beta_0 = [\beta_{0,0}, \beta_{0,1}, \ldots, \beta_{0,p}] \); \( \beta_1 = [\beta_{1,0}, \beta_{1,1}, \ldots, \beta_{1,p}] \); and \( \beta_2 = [\beta_{2,0}, \beta_{2,1}, \ldots, \beta_{2,p}] \). In (20), testing the linearity hypothesis is equivalent to prove that \( \beta_1 = 0 \) and \( \beta_2 = 0 \).

According to Teräsvirta (1994), the recommended procedure consists of an \( F \)-test with the following sequence: i) estimate the model under the assumption of linearity and compute the residual sum square \( \text{RSS}_0 \); ii) estimate the auxiliary regression (20) to obtain \( \text{RSS}_1 \); and iii) compute the critical value of the \( LM \) statistics:

\[
LM = \frac{\left( \text{RSS}_0 - \text{RSS}_1 \right)}{\text{RSS}_0} \frac{1}{l_1} \frac{1}{l_2}
\]  

(21)

The \( l_1 \) degrees of freedom are calculated as the difference between the number of parameters in the unrestricted and restricted models, whereas \( l_2 \) is calculated as the number of observations minus the unrestricted model parameters. If the linearity test is performed on a list of possible transition variables, it will be necessary to define which variables will be considered. In such a case the highest \( LM \) statistics is employed to select the transition variable.

The third step is the estimation of the STAR model, which can be done by any conventional non-linear method. This will require the definition of initial conditions. An appropriate selection of these conditions will increase the probability of reaching a maximum in the likelihood function.

The usual practice to find initial conditions is conducting a two dimensional grid search on parameters \( \gamma \) and \( c \). It is important to note in this regard that once the values of both parameters are fixed, the function \( F(TV_{t-d}; \gamma, c) \) will lie in the interval \([0, 1]\) in each observation, being equation (11) linear in all its arguments. The grid search iterates different values of \( \gamma \) and \( c \) taken at intervals that are relevant in accordance with their respective scales of variability. The conditional estimation can be done by a linear method as OLS or SUR. The configuration of \((\gamma, c)\) which generates the restricted model with the
maximum likelihood will be selected. Then, its parameters are used as initial conditions to estimate the unrestricted STAR model.

Theory could also provide some conditions for the smoothing parameter ($\gamma$) or the threshold ($c$). In those models where the misalignment is the state variable ($TV_{t-d} = M_{t-d}$), the condition $c = 0$ is often imposed because of the symmetry in the economic concept of misalignment itself. Thus, when the differences between actual and fundamental prices are very small, then $F(TV_{t-d} : \gamma, c = 0) = 1 - \exp\left(-\gamma (M_{t-d})^2 \right) \rightarrow 0$, and so we will observe the system operating in one of the extreme regimes. In papers about purchasing power parity, transaction and transportation costs limit arbitrage and thus small mismatches are interpreted as a state variable, where PPP do not hold. The corollary is that the exchange rate behaves like a random walk if $M_{t-d}$ is low.

Finally, a useful tool in a STAR model applied only to systems of equations is the computation of generalized impulse-response functions.

A methodological description of these impulse-response functions is beyond the scope of this paper. However, the intuitive idea is that in a STAR model the effects of shocks depend on the history, size and sign of disturbances. For instance, the effect of a shock will not be necessarily the same if the shock occurs when there is a small misalignment with respect to the case of a high initial gap between current and equilibrium commodity prices. Also the size of shock could be relevant since it could involve different future dynamic trajectories of the endogenous variables. These characteristics are not incorporated by traditional impulse-response functions estimation methods.

We follow the bootstrapping methodology of Koop et al. (1996) to compute the generalized non-linear impulse-response functions. We suggest reviewing this reference for a complete discussion of technical details.
6. Non-linearity in the adjustment of commodity prices: empirical results

With the aim of organizing the presentation of the empirical model, this section has been divided into four parts. In the first part, the variables and the data sources are described. Subsequently, we discuss the estimates of the long-run equilibrium equation of commodity prices and show the time path of the implied misalignment. In the third sub-section we present the non-linearity tests results, the interpretation of the transition function and its regimes, and the estimation of the non-linear system. Finally, impulse-response analysis is performed to investigate the short-term reaction of commodity prices to shocks in fundamentals under both high and low misalignment regimes.

6.1 The variables of the empirical model

Our empirical analysis is based on monthly data belonging to the post Bretton Woods era, from 1973M01 to 2008M05.

After defining the period of analysis, we have to choose the commodities whose prices will be explained. Given that our theoretical framework is general enough and holds for a broad set of assets, we opt for studying an aggregated index. Particularly, the Food Index and the Metals Index from the International Finance Statistics (IFS) were averaged out to construct the All-Comm Index. To deflate the series, US CPI from the same source was used.\textsuperscript{8} Figure 13 shows the evolution of the index in both nominal and real terms.\textsuperscript{9}

\textsuperscript{8}It is worth mentioning that oil was not considered because several authors such as Beenstock (1988) or, more recently, Cheung and Morin (2007) have stressed it has its own dynamic with low connection to other commodities.

\textsuperscript{9}A clarification related to the issue of commodities as financial assets is needed. Price indexes we use are elaborated with data from spot prices and not from futures or other similar derivative instruments, which are in strict financial terms the real investment vehicles. However, the argument is that spot prices are very good indicators of the financial returns of commodities. In this sense, Gorton and Rouwenhorst (2004) show that the return on a future position and the current price has a correlation close to one.
As it was highlighted in the section one, the outlook for commodities dramatically changes whether nominal or real indexes are considered. Both series have exhibited an important growth since 2002; while the former has increased 166.3% up to 2008 from then, the latter has risen 124.3% in the same time interval. However, the overall performance has been very different: whereas in the first semester of 2008 the nominal index averaged a value 2.6 times higher than the figure of 1973, current real prices are roughly a half of the value observed at the beginning of the sample.

Regarding price fundamentals definition, we have the US real exchange rate ($RER$) in the first place. The broad multilateral version published by the Federal Reserve Bank of New York is employed. The variable is defined in such a way that it rises when the dollar is appreciating against the currencies of main trading partners.

As real international interest rate ($IR$) is utilized the one-year Treasury constant maturity rate from the Board of Governors of the Federal Reserve System. Again, US CPI is the deflator.

The price fundamental whose measurement presents serious difficulties is the commodity world demand proxy, especially because a series in monthly basis is required. Industrial Production of the developed...
countries is the alternative usually employed in studies on this topic. However, Asian emerging countries have become crucial players in commodity markets and they should be incorporated into the analysis. Therefore, we consider an Industrial Production Index (PRO) which also takes into account the industrial output of Korea, India, Malaysia and China. All the indexes come from the IFS, except from China whose series is built using the Industrial Value Added provided in an annual-basis by the IMF World Economic Outlook and linearly extrapolating to become it monthly. We use the share of each country in the industrial added value (IVA) as a weighting factor. The seasonally unadjusted series is corrected applying the X-12 ARIMA method.

In addition to the price determinants reviewed in the first section, the Dow Jones index deflated by US CPI (DOW) is also included. Our aim is to control for the return of alternative assets and to investigate if conditional covariance indicates substitution or complementary effects between these asset classes. As it was mentioned before, authors like Gorton and Rouwenhorst (2004) or Deutsche Bank (2005) have studied this link and they have found that the non-conditional correlation between returns of commodities and other assets (bonds and equities) is negative and very significant in the long term. In Domanski and Healt (2007) the ratio of net long contracts for non-commercial agents is negatively correlated, though slightly, with stock indexes.

6.2 The structure of the model, equilibrium estimation and the misalignment

The structure of the model to be estimated is similar to a VECM but includes non-linear terms in the commodity price equation. In this sense, we move away from the seminal work of Weize (1999) that specifies a symmetrical smooth transition VAR; which means all the equations present non-linear terms. Our decision is based on the fact we only have a theoretical rationale for non-linear adjustment in the price

---

10The weights we obtained by this criterion were increased by 50% in the case of emerging countries for two reasons. The first one is that emerging economies are underrepresented in the sample. The second reason is that these countries have a greater intensity of commodity consumption per unit of output. Capturing a specific amount of increase of this coefficient is beyond the scope of this study, but increasing it seems a better alternative than to ignore the two above-mentioned effects.
dynamics. An alternative empirical strategy could have been using a single-equation STAR.\footnote{This is the strategy adopted by Westerhoff and Reitz (2005) to explain mean reversion in corn prices. Other difference of that model in comparison to our empirical framework is these authors do not use an exponential specification for the transition function.} However, that option would have implied abandoning the possibility of studying the short-term commodity price response to shocks in fundamentals and we would not be able to study interactive effects.

In particular, we take (19) as a benchmark and work with the following system:

\[
\Delta P_t = \beta_{11} + \alpha_{11} (M_{t-1}) + \sum_{j=1}^{p} \Pi_{11,j} \Delta X_{t-p} + \\
\left( \beta_{12} + \alpha_{12} (M_{t-1}) + \sum_{j=1}^{p} \Pi_{12,j} \Delta X_{t-p} \right) \left( 1 - \exp(-\gamma TV_{t-d}^2) \right) + \epsilon_t^P
\]  

(22)

\[
\Delta RER_t = \beta_{2} + \alpha_{2} (M_{t-1}) + \sum_{j=1}^{p} \Pi_{2,j} \Delta X_{t-p} + \epsilon_t^{RER}
\]  

(23)

\[
\Delta IR_t = \beta_{3} + \alpha_{3} (M_{t-1}) + \sum_{j=1}^{p} \Pi_{3,j} \Delta X_{t-p} + \epsilon_t^{IR}
\]  

(24)

\[
\Delta PRO_t = \beta_{4} + \alpha_{4} (M_{t-1}) + \sum_{j=1}^{p} \Pi_{4,j} \Delta X_{t-p} + \epsilon_t^{PRO}
\]  

(25)

\[
\Delta DOW_t = \beta_{5} + \alpha_{5} (M_{t-1}) + \sum_{j=1}^{p} \Pi_{5,j} \Delta X_{t-p} + \epsilon_t^{DOW}
\]  

(26)

Where \( X = [P, RER, IR, PRO, DOW] \) and the threshold coefficient \( c \) is set equal to zero because of theoretical reasons discussed in Section 5. As before, \( M_t = P_t - F(X_t) \) and \( TV_{t-d} \) is the transition variable.

The main focus of the empirical analysis is centered on the expression (22) as it contains the price dynamics which is similar to that derived from the theoretical model (equation (9)). Particular interest should be given to the estimates of \( \alpha_{11} \) and \( \alpha_{12} \) which represent the price adjustment coefficients to the deviations from the long-term equilibrium.
The econometric strategy adopted is in line with the Engle and Granger proposal. In essence, we estimate in a first stage the long run equation for commodity prices and test for cointegration. Then, if a cointegration relationship is found, the whole error correction system will be estimated using $M_t = P_t - F(X_t)$ as a regressor.

To carry out the first stage requires coping with non stationary series.12 As it is well known, if I(1) variables are cointegrated the OLS estimator of the coefficients of the long run relationship will be consistent. However, it will have a non-normal distribution, and inferences based on $t$-statistics will be misleading.

Several econometrics techniques have been developed to overcome this problem. We apply the Dynamic OLS (DOLS) estimator developed by Stock and Watson (1993) which adds leads and lags of the changes in the right hand regressors in the standard long run equation. That is,

$$P_t = \beta_1 + \beta_2 X_t + \sum_{j=-p}^{p} \beta_3 \Delta X_{t-p} + \beta_4 t + \epsilon_t$$

(27)

Where $X_t$ stands for the price determinants and $p$ represents the number of leads and lags considered. DOLS estimator of $\beta_2$ results from OLS estimation of equation (27).

If the variables are cointegrated, the DOLS estimator is consistent and efficient in large samples. The methodology deals with potential simultaneity and small sample bias among the regressors by the inclusion of leads and lags. Besides, Monte Carlo experiments show that DOLS performs better, particularly in small samples, compared to alternative estimators of long-run parameters as those proposed by Engle and Granger (1987), Johansen (1988), and Phillips and Hansen (1990).13 Finally, standard statistical inference remains valid when heteroskedastic and autocorrelation consistent (HAC) standard errors are employed.

12The usual unit root tests were implemented confirming all the series are non stationary. Results are available upon request to the authors.

13See Stock and Watson (1993) or Montalvo (1995) It is worth mentioning that DOLS estimator is asymptotically equivalent to the Johansen estimator (Stock and Watson, 1993).
We apply this methodology to obtain the coefficients of the long run equation of commodity prices (Table 2). We use six leads and lags but the estimation was robust to changes in the value of $p$.

Table 2. DOLS Estimates of long run equation

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS Coefficient</th>
<th>DOLS Coefficient</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial Production Index</td>
<td>3.6124</td>
<td>4.1368</td>
<td>0.0000</td>
</tr>
<tr>
<td>US Real Exchange Rate</td>
<td>-0.7270</td>
<td>-0.5994</td>
<td>0.0000</td>
</tr>
<tr>
<td>Real International Interest Rate</td>
<td>-1.0907</td>
<td>-1.5452</td>
<td>0.0138</td>
</tr>
<tr>
<td>Real Dow Jones Index</td>
<td>-0.1299</td>
<td>-0.1086</td>
<td>0.0163</td>
</tr>
<tr>
<td>Time Trend</td>
<td>-0.0097</td>
<td>-0.0111</td>
<td>0.0000</td>
</tr>
<tr>
<td>Constant</td>
<td>-4.1659</td>
<td>-6.9366</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Before analyzing these results it is necessary to perform a cointegration test. If there exists no cointegration among variables, it is not possible to evaluate the estimates because of the spurious regression problem.

As it is remarked by Choi et al. (2008), DOLS is employed in many applications but few cointegration tests have been developed for it. These authors propose a Hausman-type test but it does not allow for regressions with time trends as is the case of our model. Therefore, we use the test proposed by Shin (1994) which admits all kind of deterministic components.

An interesting common feature of both tests is that their null hypothesis is the presence of cointegration, in contrast to the no cointegration null hypothesis of standard ADF-type tests. Shin (1994) and Ogaki and Park (1998) argue that cointegration is the desirable null hypothesis in several applications.

The Shin cointegration test statistic when a constant and time trend are present in the long run equation takes the following expression
\[ C_\tau = T^{-2} \sum_{t=1}^{T} S_t^2 / s_\tau^2(l) \]  

(28)

Where \( S_t = \sum_{j=1}^{l} \epsilon_j \), \( \epsilon_j \) are the DOLS residuals from equation (27), and \( T \) stands for the sample size.

Finally, \( s_\tau^2(l) \) is the semiparametric consistent estimator of the long run variance of \( \epsilon_j \).

We find a value of 0.07796 for \( C_\tau \) which is lower than the 2.5% critical value (0.088).\(^{14}\) Thus, there is evidence of cointegration among commodity prices and their determinants.\(^{15}\) Then, we can analyze the results showed in Table 2.

In the first place, it is remarkable that all the variables are statistically significant and their signs are in line with the theoretical predictions.

Next, regarding the value of the coefficients, we observe that a real devaluation (fall) of the dollar by 1% implies a 0.60% increase in commodity prices. Thus, this elasticity is lower than one in absolute value as suggested by the Ridle and Yandle (1972) model. It is also relevant to highlight the high elasticity of commodity prices to industrial production (4.13%).

Stock market performance impacts negatively on the price of commodities. Along the findings by Domanski and Healt (2007), the substitution effect between the two asset classes tends to predominate. As for the coefficient of the real interest rate, it is slightly bigger than -1 in absolute value (-1.09).

Finally, the negative sign of the time trend supports the Prebisch-Singer hypothesis. This result corroborates the first sight intuition suggested by Figure 13, where declining real prices were observed.

In Figure 14, actual and “equilibrium” price series are depicted while implied misalignment is portrayed in Figure 15.

\(^{14}\)Shin (1994) provides the critical values for his cointegration test statistics.

\(^{15}\)For robustness check, we carried out the standard ADF non-cointegration test based on OLS residuals and we rejected the null hypothesis of unit root residuals.
Figure 14. Commodity price index: actual values and equilibrium (logarithmic scale)

Figure 15. Commodity prices: estimated misalignment
From both figures we emphasize the fact that the magnitude of the misalignments observed during 2006-2007 are similar to those observed in the previous years to the 80s debt crisis and both Asian and Russian Crisis (roughly 20-25%).

In the next section we will see how misalignment plays a dual role: as an adjustment variable of short term deviations, and as a potential determinant of the state.

6.3 The short-term model and nonlinear equilibrium adjustment

In the first place, as it was stated in Section 5, testing the nonlinear equilibrium adjustment hypothesis requires estimating a linear model which acts as benchmark.

To this end, we specify a linear symmetrical vector error correction model (VECM) using the previously calculated misalignment. The Akaike Information Criterion (AIC) suggests using five lags for the changes in the variables. In addition, under this specification the residuals do not present autocorrelation.16

The adjustment coefficient \( \alpha_{11} \) in equation (22) takes a value of -0.01921 and is statistically significant at 10% when Newey-West HAC standard errors are considered. Thus, there is evidence that, at least in average, correcting forces emerge when commodity prices are in disequilibrium.

However, as our theoretical model of section 4 predicts, it is really possible that there exist some states where misalignments are high and so correcting forces operate, while in other states characterized by low misalignments the gaps remain. This is the intuition behind the non-linear equilibrium adjustment hypothesis.

Once we have estimated the benchmark linear model, we carry out the linearity F-test on the commodity price equation as it was described in the methodological section. The auxiliary regression takes the specific following form:

16We performed the Serial Correlation LM test with 12 lags and we failed to reject the null hypothesis of no autocorrelation at 5% statistical significance.
\[ \Delta P_t = \beta_{0t}p\Delta X_{t-p} + \nu_1 M_{t-1} + \beta_{1t}p\Delta X_{t-p} TV_{t-d} + \nu_2 M_{t-1} TV_{t-d} + \beta_{2t}p\Delta X_{t-p} TV^2_{t-d} + \nu_3 M_{t-1} TV^2_{t-d} + \omega_t \quad (29) \]

Where \( \Delta X = [I, \Delta P_{t-p}, \Delta RER_{t-p}, \Delta IR_{t-p}, \Delta PRO_{t-p}, \Delta DOW_{t-p}] \). \( TV_{t-d} \) stands for the transition variable, and \( p : l,...,5 \). Note the first two terms in equation (29) correspond to the linear specification of the commodity price equation (22) in the VECM model described above.

As we have already stressed, our theoretical model suggests the regimes are governed by the size of misalignment. Thus, we propose as potentially transition variables five ones. One of them is simply the misalignment \( (M_{t-d}) \) and the others result from averaging out the current misalignment and those of the previous \( j \) periods \( (AV_{j-t-d}) \), where \( j \) ranges from 1 to 4. Besides, we consider \( d = l,...,12 \) lags for each of them.\(^{17}\)

Table 3 shows the test results for the ten variables which exhibit the lowest p-values.\(^{18}\) Remember that the null hypothesis is linear adjustment in the commodity price equation (22).

<table>
<thead>
<tr>
<th>Transition Variable</th>
<th>Lag</th>
<th>LM Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV1</td>
<td>1</td>
<td>1.38647</td>
<td>0.04912</td>
</tr>
<tr>
<td>AV4</td>
<td>12</td>
<td>1.38309</td>
<td>0.05040</td>
</tr>
<tr>
<td>AV3</td>
<td>12</td>
<td>1.21178</td>
<td>0.16396</td>
</tr>
<tr>
<td>AV4</td>
<td>11</td>
<td>1.19485</td>
<td>0.18166</td>
</tr>
<tr>
<td>AV1</td>
<td>6</td>
<td>1.15964</td>
<td>0.22287</td>
</tr>
<tr>
<td>AV2</td>
<td>1</td>
<td>1.15512</td>
<td>0.22859</td>
</tr>
<tr>
<td>AV2</td>
<td>12</td>
<td>1.12511</td>
<td>0.26910</td>
</tr>
<tr>
<td>AV2</td>
<td>6</td>
<td>1.12423</td>
<td>0.27036</td>
</tr>
<tr>
<td>AV4</td>
<td>4</td>
<td>1.12396</td>
<td>0.27074</td>
</tr>
<tr>
<td>AV3</td>
<td>5</td>
<td>1.12106</td>
<td>0.27490</td>
</tr>
</tbody>
</table>

\(^{17}\)We do not test \( TV_{t-1} = M_{t-1} \) because of perfect multicollinearity problem when system (22)-(26) is estimated.

\(^{18}\)This test was programmed on E-views. The code is available upon request.
We find evidence of non-linearity in equation (22) at 5% statically significance when the variable $AV_{t-1}$ is considered. That is, regime in each moment $t$ would be defined by the average value of the misalignments registered in $t-1$ and $t-2$.

Therefore, we will estimate the non-linear system (22)-(26) considering $AV_{t-1}$ as transition variable. In the Section 5 we have anticipated it is necessary for this to provide proper initial conditions for all the coefficients of the system.

Those conditions can be found estimating the linear system which is obtained after fixing the value of the parameter $\gamma$ in equation (22). Instead of carrying out a grid search, we select a value of $\gamma$ such that the transition function averages 0.5 in the whole sample. Intuitively, this means that portfolio managers assign the same weight to both fundamentalist and chartist agent expectations in average. Consequently, initial value of $\gamma$ approximately should satisfy the following expression:

$$w_t = \frac{1}{T} \sum_{i=1}^{T} \left( 1 - \exp \left( -\gamma AV_{t-i} \right) \right) \approx \frac{1}{2} \tag{30}$$

We find that $\gamma = 133.77$ numerically solves (30) for the 1973M01-2008M05 period. Then, after setting $\gamma$ at this value, the system (22)-(26) is estimated by SUR and initials conditions for all the coefficients are obtained.

Once we have these conditions, we proceed estimating the unrestricted non-linear system also by SUR.\(^{19}\)

In order to evaluate our hypothesis, analysis is centered in the adjustment coefficients of the commodity price equation, namely $\alpha_{11}$ and $\alpha_{12}$.

The linear error correction coefficient $\alpha_{11}$ appears not statistically significant (p-value=0.2914) and takes a positive value of 0.035. Contrary, the non-linear adjustment coefficient $\alpha_{12}$ results significant at 5% (p-

\(^{19}\)The E-Views code used in this section is also available upon request to the authors.
value=0.0418) and is equal to -0.109. Moreover, the $\gamma$ parameter reaches a magnitude of 55.22 with a p-value equal to 0.0292.

These estimations support our non-linear adjustment hypothesis due to heterogeneous agents in commodity markets.\textsuperscript{20} To see this, we define the Global Equilibrium Correction Factor (GEC) as the two terms of the price equation (22) containing the price deviation from its long run equilibrium ($M_{t-1}$).

$$GEC = (0.035)M_{t-1} + (-0.109)M_{t-1} \cdot w_t = \{(0.035) + (-0.109)w_t\}M_{t-1} \quad (31)$$

When misalignment is low enough (low regime), $w_t$ tends to zero and the second term of (31) vanishes. In this case, portfolio managers mimic chartist investors and initial misalignment is widened at a 3.5% monthly rate. Under these circumstances, there is no equilibrium correction.

However, when the gap between actual and equilibrium prices reach a sizable value (high regime), $w_t$ tends to one and the GEC adjustment coefficient attains a maximum of -0.074 ($=\alpha_{11} + \alpha_{12}$). Intuitively, portfolio managers assign a larger weight to fundamentalist investor expectations when misalignment is high and therefore we will observe price reversion toward the equilibrium.$^{21}$

A key question is to determine what high and low commodity price misalignment means in our model. Figure 16 provides information to answer it.

\textsuperscript{20}Most of the arguments will be stated considering positive misalignments, but the same applies for negative gaps because of the symmetry of exponential function.

\textsuperscript{21}Given that non-linear adjustment coefficient ($\alpha_{12}$) is negative and larger than $\alpha_{11}$ in absolute terms, the overall stability of the price equation is guaranteed.
The LHS panel of Figure 16 reveals that price correction forces prevail in the market only when gap is larger than 8.5%. Considering the transition variable function (RHS panel), adjustment to equilibrium predominates when TVF (and \( w_t \)) exceeds 0.32. Furthermore, both panels show that non-linearity implies bigger adjustment coefficients, the higher the misalignment is. Maximum reversion speed (-7.4% monthly) is attained when spread between actual and fundamental commodity prices surpasses 25%.

The corollary is that higher misalignments in the past (measured by \( AV_{t-1} \)) involve higher values of the transition function and this will indicate stronger future trend reversals.

Finally, Figure 17 depicts the transition function in the period considered.
In order to analyze the distribution of the states, we will assign observations to low regime if transition function takes values lower than 0.30 whereas we will consider they belong to high regime every time transition function exceeds 0.70.\textsuperscript{22} The remaining observations form the “transition regime”. Given this criterion, we find 56% of the time the market is dominated by chartists while fundamentalists only prevail about 18.85%. The rest of the time (25%) corresponds to transition periods.

\textsuperscript{22}These figures imply that global adjustment coefficient is positive in the low state and larger in absolute value than -4% in the high state.
Transition function averages out 0.34 in the whole sample. Therefore, mean price change expected by portfolio managers is as follows

\[
E(\Delta P_{PM}^{t+1}) = (1 - \bar{w}_t)E(\Delta P_{C}^{t+1}) + \bar{w}_tE(\Delta P_{F}^{t+1}) = 0.66E(\Delta P_{C}^{t+1}) + 0.34E(\Delta P_{F}^{t+1})
\]  
(32)

The \textit{GEC} coefficient associated to this figure is just -0.18%.

Summarizing, results support the hypothesis that high discrepancies between actual and equilibrium prices tend to be corrected relatively fast, while small misalignments tend to persist over time without any endogenous correcting force taking place.

In the next section we perform impulse-response analysis following the theoretical guidelines exposed in the fifth section.

\textbf{6.4 Non-linear impulse-response analysis}

As it was stressed in the methodological section, in an exponential STAR model the effects of shocks depend on the history and size of disturbances.\textsuperscript{23} These properties are not incorporated in traditional impulse-response analysis.

Before presenting the results we will indicate just two technical issues. In the first place, to carry out the generalized impulse-response function of Koop et al. (1996) requires assigning each observation to different regimes. We do this adopting the same allocation criterion stated in the previous section. Then, sampling of shocks is performed.

Secondly, initial state is defined by the analyst but after shock takes place, it could endogenously change as consequence of the system dynamics.

Figures 18 to 21 show the accumulated change of commodity prices to one-standard deviation shock in the fundamentals for both high and low initial misalignment states. In the Appendix the effects of three-standard deviation shocks are depicted. The dashed lines are the 10% confidence bands.

\textsuperscript{23}This is the case when the transition function is an exponential function. If we had selected a logistic specification, the sign of the shock would also matter.
Figure 18. US effective real exchange rate shock

Figure 19. Industrial production shock
The main conclusion we could draw from the figures is that, in general, the responses obtained are in line with both the theoretical predictions and the long-run relationship estimation. The only exception is that we find a positive impact of the Dow Jones index in the short run.
Furthermore, it is noteworthy that price dynamics responses do not exhibit significant differences either the initial state is low or high. The effect of real interest rate shock differs from this general picture. If the shock occurs in the high regime, the commodity price reaction is minor and statistically non significant. The same remarks apply when we study the consequences of bigger shocks (three-standard deviation). There are not important discrepancies when the effects of small and big shocks are compared, except for the interest rate shock response which becomes statistically significant whatever the initial regime is.

7. Conclusions and policy implications

From the policymaker perspective, the distinction between permanent and transitory movements in macroeconomic variables is one of the main challenges in order to take proper economic decisions. An important objective of applied economic models and empirical estimations is trying to give a consistent framework to rationalize those decisions. With this aim, we have proposed employing a non-linear multivariate STAR methodology to reach a better understanding of underlying causes for commodity price movements, once an explicit role for the financialization issue is incorporated.

It is appealing to think financialization as an amplifying factor of commodity price cycles. We develop a framework in which fundamentals and financialization interact each other, treating speculative activity as an element that mainly affects short run price dynamics, but not the long run equilibrium.

This hypothesis appears satisfactory after summarizing the discussion regarding the financialization issue and showing evidence that it is not necessary to have neither commodity derivative markets nor strong net long positions of financial participants to experience a commodity price boom or a bust.

Thus, in our theoretical model, commodity fundamentals continue to be the only real force to forecast long run prices. However, heterogeneity in expectations among commodity market participants is important in determining the characteristics of the equilibrium adjustment.

Regarding fundamentals we have empirically confirmed some standard roles for macroeconomic variables, namely US real exchange rate, aggregated industrial production as a proxy of world demand,
and real international interest rate. We also verify a negative conditional correlation between our real commodity price index and the real return of stock markets.

The short run dynamics shows the most remarkable results. Our findings support the idea that commodity prices tend to correct toward equilibrium, but this correction only takes place if past misalignment is sufficiently high. Thus, in the low misalignment regime correcting forces do not prevail and prices can move in any direction, possibly depending on market sentiments.

Finally, regarding policy implications, we want to point out that for commodity-dependent developing countries, commodity price misalignments should be carefully monitored: price reversions tend to be abrupt when the gap between actual and fundamental price is higher than 20-25%. We cannot forget that commodities shape almost every macroeconomic policy stance in these countries: from output growth and inflation to income distribution, national savings or fiscal revenues.

It is also important to note that factors affecting commodity prices (like real international interest rates and the US real exchange rate) are similar to those that influence capital flows. This explains why it is hard for developing countries to cushion terms of trade shocks with external finance. The same fundamentals that worsen terms of trade affect negatively the access to international credit market. An appropriate policy mix should include in this case structural actions to smooth external cycles and alleviate commodity dependence when prices are in high levels.
References


Appendix. Impulse-response analyses (three-standard deviation shocks)

Figure 22. US Effective real exchange rate shock

Figure 23. Industrial production shock
Figure 24. US real interest rate shock

a) Low misalignment state                  b) High misalignment state

Figure 25. Real Dow Jones index shock

a) Low misalignment state                  b) High misalignment state